Technical Document Series

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M&CT-TECH-02-004

March 2002

Mathematics and Computing Technology
Phantom Works

Phantom Works, Mathematics & Computing Technology

A Division of The Boeing Company

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A Comparison of Interior Point and SQP Methods on Optimal Control Problems

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March 11, 2002

Abstract

This paper compares four different methods to solve sparse *Nonlinear Programming Problem*'s (NLP)'s arising out of the discretization of optimal control problems. The four methods compared are SNOPT, SPRNLP, LOQO and BARNLP. Of these the first two are SQP methods, whereas the other two are interior point methods. A set of 173 test problems varying both in size and difficulty is used for this comparison.

1 Introduction

In this paper, we compare different classes and implementations of solvers for *Nonlinear Programming Problem*'s (NLP)'s. We only look at a special group of NLP's created by Boeing's *S*parse *O*ptimal *C*ontrol *S*oftware (𝕊𝕆ℂ𝕊) [8]. 𝕊𝕆ℂ𝕊 solves sparse optimal control problems by using a "*direct transcription* or *collocation method* to convert the continuous control problem into a discrete approximation" [8]. This approximation then yields a sparse NLP. In order to compare the four methods, we look at 30 different optimal control problems with different kinds of discretizations and different initial and/or boundary conditions. These different optimal control problems represent a wide range of problems, from academic to industrial problems. The resulting 173 NLP's differ in size from tens of variables and constraints to ten-thousands of variables and constraints. 𝕊𝕆ℂ𝕊 provides first and second order derivative information to the solvers by using sparse differencing, see [3]. First derivative information is calculated either with forward or central sparse differences, resulting in different accuracies and costs.

Our goal was to compare four different methods to solve these NLP's. These four methods belong to two different classes of solvers, the class of *S*equential-*Q*uadratic-*P*rogramming (SQP) methods and the class of *I*nterior *P*oint (IP) or *barrier* methods. The four implementations we used were

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- SNOPT
- SPRNLP
- LOQO
- BARNLP

Both SNOPT and SPRNLP are SQP methods. SNOPT was developed by Gill, Murray, and Saunders [11]. SPRNLP, a Boeing code, was written by Betts and Frank [6]. The two interior point methods we investigated were LOQO [14] and the Boeing code BARNLP [4]. The LOQO software implementation is described in Vanderbei [12].

2 Test problems

A collection of nonlinear programming test problems derived from optimal control applications is given in Reference [7]. These problems have been collected over more than a decade from practical applications in aerospace trajectory optimization and optimal control. The suite contains problems of different size, nonlinearity, and difficulty and have been used in previous algorithmic comparisons [5]. The suite was chosen because it is very representative of the target applications for the nonlinear optimization software.

3 Solvers

This section provides a short overview of the four different methods that are used in our comparison. Our intent is only to provide a brief overview of the differences in the methods, and we will not attempt a detailed description. We refer the reader to [14, 12, 11, 10, 13, 9, 3, 6, 4] for more information. In the following we first standardize the notation of the NLP.

All four algorithms solve the *N*onlinear *P*rogramming *P*roblem (NLP). In this problem we want to *minimize* the scalar objective function

$$f(\mathbf{x}) \tag{1}$$

subject to the m nonlinear constraints

$$\mathbf{c}_L \leq \mathbf{c}(\mathbf{x}) \leq \mathbf{c}_U, \tag{2}$$

and the simple bounds

$$\mathbf{x}_L \leq \mathbf{x} \leq \mathbf{x}_U, \tag{3}$$

where $\mathbf{x} \in \mathbb{R}^n$ [3].

Superbasics limit	4000
Iterations limit	$\max(20000, 50m)$
Minor Iterations limit	$\max(1500, 10m)$
Major Iterations limit	$\max(1000, 2m)$
Major feasibility tolerance	1.D-8
Major optimality tolerance	1.D-9
Minor feasibility tolerance	1.D-10
Minor optimality tolerance	1.D-6
Verify level	0

Table 1: Non-standard settings for SNOPT

3.1 Sequential Quadratic Programming (SQP) Methods

Sequential quadratic programming (SQP) methods solve a nonlinear programming problem (NLP) by solving a sequence of Quadratic Programming (QP) problems.

The two (SQP) methods we use differ mainly in how they

- approximate the Hessian,
- choose the step length in the line search, and
- find the solution of the (QP).

3.1.1 SNOPT

For the purpose of these computations, we used SNOPT version 5.3-4. SNOPT uses a BFGS quasi-Newton update with safeguards to maintain a positive definite Hessian approximation, see [10]. This means that SNOPT only uses first-order derivative information and cannot exploit sparsity information in the Hessian. This inability to exploit sparsity information is a major disadvantage for SNOPT when applied to large problems. Even though SNOPT uses a dense quasi-Newton Hessian, in contrast to the other algorithms which exploit a full sparse Hessian, it is included in our tests because of its widespread use.

Table 1 shows the settings we used for SNOPT which differ from the standard settings. Computations were done using both forward sparse differencing and centered sparse differencing to approximate the first derivatives. The latter provides more accuracy for the cost of extra function evaluations, and is what we used in the reported results. The higher accuracy allowed us to solve more of the test problems.

SNOPT gives the user the option to define linear parts both of the objective function, as well as the constraints. Since our NLP's were created by $\mathbb{SOCS}$, we were not able to supply this information. Therefore, by necessity, the whole objective function and all constraints were treated as nonlinear.

In addition, SNOPT allows the user to instruct the optimizer that an error occurred during the function or constraint evaluation. In spite of this feature

in the software, better results were achieved by assigning a value of 1.e42 when an error was encountered during a function evaluation. Assigning this value allowed us to solve some problems in which SNOPT stopped the optimization completely when an error in the function evaluation was reported. SNOPT encountered function errors in 23 problems, and was able to recover and solve 12 of these problems.

3.1.2 SPRNLP

SPRNLP uses first and second order information to create the quadratic approximation. If the Hessian is not positive definite, it is modified using a Levenberg parameter. Furthermore SPRNLP incorporates four general strategies. These are:

M	*Minimize*. Optimize from the start.
FM	*Feasible Minimize*. Find a feasible point and then minimize the function.
FME	*Feasible Minimize Equality*. Find a feasible point and then minimize the function subject to equalities.
F	*Feasible*. Find a feasible point.

The first strategy is the most aggressive one, since it assumes that the starting point is already close to the solution. This is very often the case when grid refinement is done, and the solution from a courser grid provides a good starting point for a finer grid. The most conservative of the three optimization strategies is the third one, which in most cases is the slowest. A good mixture of the two other strategies is the second, which is also the default for SPRNLP. It combines robustness and good performance. All numerical results in this report used the second strategy.

Finally the fourth strategy is used if no optimization is required and some problems only require finding a feasible point.

To solve the underlying systems of linear equations both Boeing codes use the efficient multifrontal algorithm [1].

3.2 Interior Point Methods

For a description of interior point methods see [14, 4].

3.2.1 LOQO

There are a number of implementation issues that affect the performance of the interior point algorithm in LOQO. For the comparisons in this paper we used version 6.02 of LOQO. Since LOQO is written in C, we had to write a C interface to our FORTRAN routines which define the problems. The interface has to be taken into account when comparing the run times, since every time LOQO needs to evaluate either the function, constraint or their derivatives, a C routine calls a FORTRAN routine. Furthermore, LOQO needs the Jacobian of the constraints in a different storage format than $\mathbb{SOCS}$ provides. This means

Maximum number of iterations	itnlim	900
Honor the bounds initially	honor_bnds_init	0
Minimum initial value for slack variables	bndpush	.1
Number of significant figures required	sf_req	9
Infeasibility tolerance	inftol	1.0e-08

Table 2: Non-standard settings for LOQO

every evaluation of the Jacobian involves copying the values of the Jacobian from one format to another.

LOQO does not take advantage of the symmetry in the Hessian, whereas $\mathbb{SOCS}$ does in that it only stores the lower triangular part of the Hessian. Therefore, every evaluation of the Hessian involves copying values from one (possibly large) array to another, in a non-sequential way.

To reduce the number of function evaluations, $\mathbb{SOCS}$ can evaluate the function and their derivatives at the same time. That is, $\mathbb{SOCS}$ can evaluate at the same time

- the objective function and the constraints, or
- the above and the first derivative of the objective function and the Jacobian of the constraints, or
- all of the above and the Hessian.

R. Vanderbei, one of the authors of LOQO, provided us with an interface for LOQO that allows us to take advantage of these savings.

LOQO, unlike the other codes used in this comparison, does not allow the function to return an error. In 14 of our test problems LOQO tried to evaluate the function where it was not defined. LOQO was able to recover from a function error and solve the optimization problem for only one of the 14 problems. Most of the time when LOQO did not find a solution it stagnated and reached the iteration limit. Even increasing the function evaluation limit did not correct this behavior.

Table 2 shows the non-standard settings we used for LOQO. We needed to increase the maximum number of iterations for several, mostly large problems. Furthermore, we increased the number of significant figures required for convergence and tightened the infeasibility tolerance to be able to create comparable results with the other methods. By asking for less accuracy, more problems could be solved. However, the goal was to compare all of the algorithms to the same level of accuracy.

4 Numerical results

We start this section by summarizing the overall results, and then proceed with a more detailed comparison of the different methods in Sections 4.2 and 4.3.

Method	Converged	Warning	Local solution	Failure
SPRNLP	164	1	8	0
BARNLP	162	4	5	2
SNOPT	137	16	6	14
LOQO	104	1	4	64

Table 3: Overall performance of the methods.

Method	Total time	Time to solve 97 problems all methods could solve
SPRNLP	6271	1928
BARNLP	12546	2297
SNOPT	1509327	1378425
LOQO	65513	4277

Table 4: Times to solve the problems

4.1 Overall results of the comparison

Table 3 shows the overall performance of the methods. The column labeled "converged" lists how many times each method converged to the best solution found by all four methods. The next column, labeled "warning", shows how many times each method found the best solution (or a feasible solution with a function value very close to the best found solution), but reported a warning. For example, the method might have stopped because of small step size, thus not satisfying its convergence criteria. Nevertheless, the best point found was feasible and the value of the function at this point was close to the minimum value found.

The fourth column lists the number of times each method converged to a local solution, that is a feasible solution whose function value is larger then the best function value found by more then 1.0e-4.

Finally the last column lists the number of times each method failed to find a solution. This could mean that the method could not find a feasible point, or did not converge.

This table shows that the two Boeing methods are very robust, and can solve a very high percentage of all problems. SNOPT is also able to solve a high percentage of the problems, although it creates more warning messages then all other methods. Finally, LOQO fails on nearly one-third of the problems. Later on we will go into more details and show that for certain classes of problems LOQO is a very efficient solver.

In Table 4 we show both the total time needed for all 173 problems (this includes failures to converge), and the time needed to solve a subset of 97 problems where all methods found similar solutions. These timings were done on a

143 Mhz Sun Ultra-1 workstation with 128 MByte of RAM, running Solaris 8.

It is clear from this table that the two Boeing methods perform similarly on the 97 problems all methods could solve, but overall the SQP method is still nearly twice as fast as the barrier method. LOQO performs well on the subset of problems all methods could solve, but the problems it fails to solve take a long time. SNOPT cannot compete with the other methods when we look at this group of problems. This is not surprising, since some of the problems are very large and SNOPT cannot take advantage of the sparsity of the matrices and does not use second order derivative information.

4.2 Detailed results of the comparison

In this section we take a more detailed look at the results. Before we do this, we explain how we compare different solutions. While some of the comparisons are one to one, in other cases, we compare all four methods with each other. We used the results calculated by SPRNLP as the reference solution, even if another algorithm produced a "better" solution, that is a feasible point whose function value is lower by more than 1.e-4 than the lowest (feasible) function value found by SPRNLP.

When comparing function values, we give the order of magnitude of the relative differences. That is, when comparing f^1 and f^2, using f^2 as the reference value, we look at

$$r_f = \left\lfloor \log_{10} \frac{|f^1 - f^2|}{\max\{|f^2|, 1\}} \right\rfloor. \tag{4}$$

Similarly, when comparing two points x^1 and x^2, using x^2 as a reference value, we look at

$$r_x = \left\lfloor \log_{10} \frac{\|x^1 - x^2\|_\infty}{\max\{\|x^2\|_\infty, 1\}} \right\rfloor. \tag{5}$$

We used the infinity norm to measure the maximum discrepancy in the coordinates.

4.2.1 LOQO versus SPRNLP

Figures 1 through 3 compare LOQO's solutions with the reference SPRNLP solutions. In Figure 1 we illustrate how well the objective function values compare between the two algorithms. Specifically, on the vertical axis we plot the relative error in the objective function values r_f. On the horizontal axis we plot the percentage of the total test suite that agree to various levels. Thus for approximately 30% of the problems in the test set, the optimal objective values agreed to within 10^{-10}. If we "step up" and ask how many problems agree to 10^{-8} we see that nearly 60% compare to this level. Within each "step" we display a second piece of information, namely the relative accuracy in the variables r_x. The darker shading corresponds to solutions with better variable

Figure 1: Comparison between LOQO and SPRNLP, focus on objective function value.

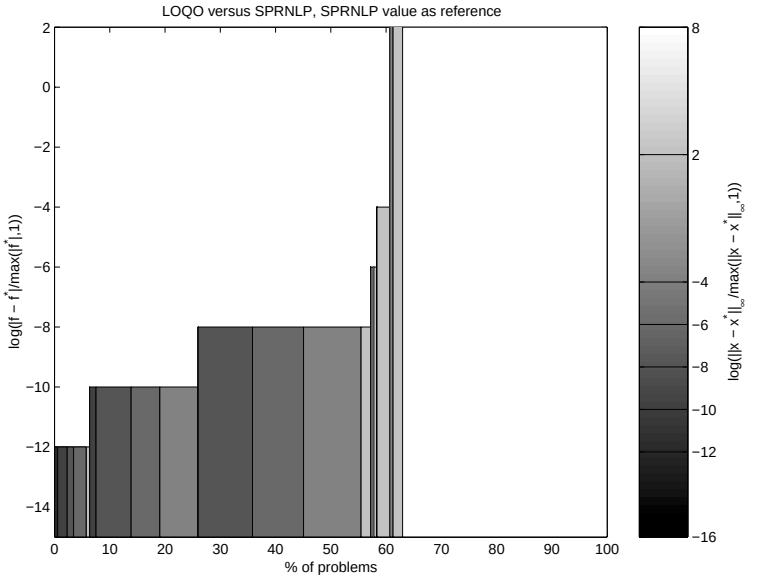


Figure 2: Comparison between LOQO and SPRNLP, focus on coordinates of the solution.

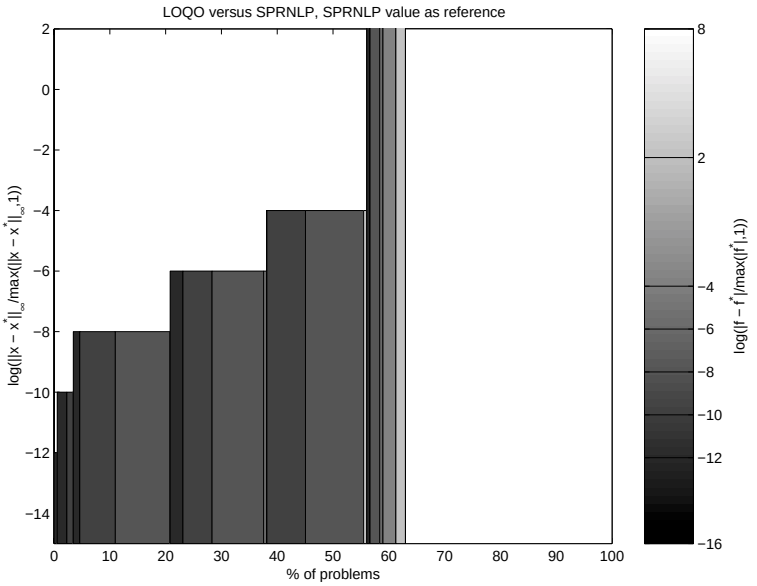
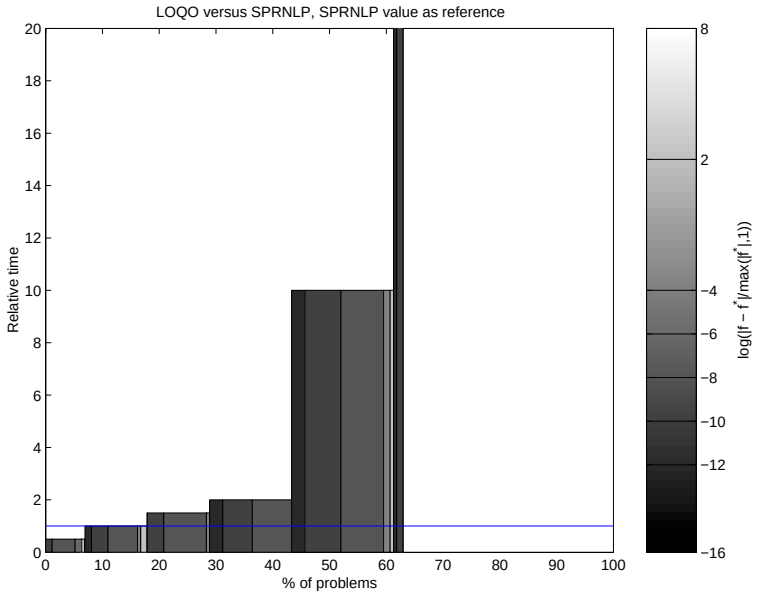


Figure 3: Comparison between LOQO and SPRNLP, focus on the solution time.



accuracy. Problems are grouped such that for all problems in the first group, the value of r_f was less than or equal to -12. In the second group the value of r_f lies between -12 and -10, and so on. The white block on the right represents the problems were LOQO did not find a point that satisfies either the nonlinear constraints or the bounds on the variables to within convergence tolerance. In this case approximately 36% of the problems were not solved by LOQO.

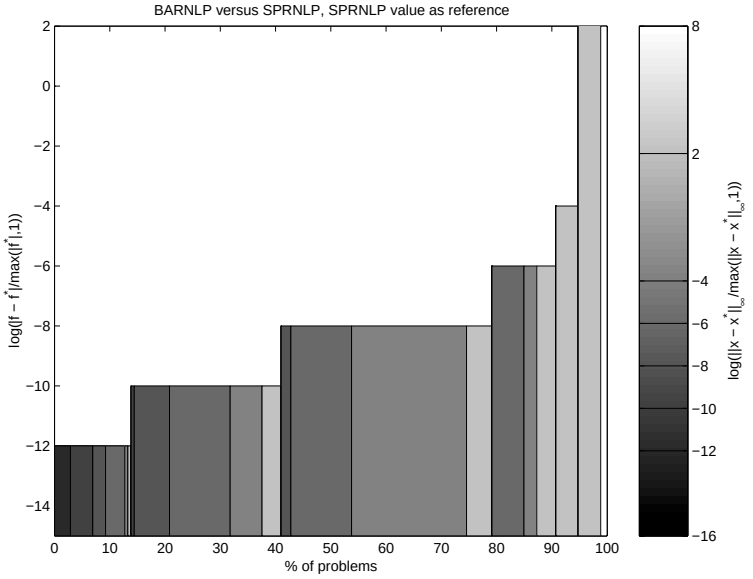
In Figure 2 we interchange r_f and r_x. That means now the y-axis measures the agreement in the coordinates, whereas the darkness measures the agreement in the objective function value.

Finally, Figure 3 compares the relative solution times for the two different methods. Note that for nearly 20 % of the problems LOQO is at least as fast or faster than SPRNLP.

Note again that we use the solution from SPRNLP as our basis for this comparison, even if SPRNLP only found a local solution. As seen in Table 3, this happens only eight times, a low percentage of all problems. This means that some of the times when the solution of SPRNLP and LOQO differs greatly, LOQO's solution is actually better.

The large percentage of failures prevent LOQO from being really competitive with the other methods. In a recent study [2] the authors of LOQO report that for equality constraint NLP's the default initialization parameters for LOQO often fail, and further research in this area is needed. For use in an industrial setting LOQO needs to include methods to handle function errors. When LOQO solves a problem, it often does so in an efficient way.

Figure 4: Comparison between BARNLP and SPRNLP, focus on objective function value.



4.2.2 BARNLP versus SPRNLP

Figures 4 through 6 compare the barrier code BARNLP and SPRNLP, using the same series of graphs as we did for LOQO. From Figure 4 it is clear that the optimal function values from these two methods agree to a high precision for nearly 80% of all problems. But, as is emphasized in Figure 5, this does not mean that the two methods find the same solution point. Instead, the solution can differ considerably, even though the function value is nearly the same. One reason for this could be that the optimal solution is very insensitive to changes, in other words, the optimum is in a flat valley. Finally, Figure 6 shows that the barrier method is as fast, or faster than the SPRNLP method for about 20% of the problems. And for about 50% of the problems BARNLP needs, at most, twice as long to obtain a solution. However for nearly half the test set BARNLP is considerably slower than SPRNLP. In Section 4.3.2 we come back to this and show that, as expected, certain kinds of problems seem to be better suited for interior point methods while others are better suited for SQP methods.

4.2.3 SNOPT versus SPRNLP

Finally Figures 7 through 9 compare the SNOPT code and SPRNLP, using the same series of graphs as before. Figure 7 and 8 show that the function value of the SNOPT solution agrees to a high precision with the function value of the SPRNLP solution, although the coordinates of the solution may differ considerably. As pointed out earlier, SNOPT only uses first order information,

Figure 5: Comparison between BARNLP and SPRNLP, focus on coordinates of the solution.

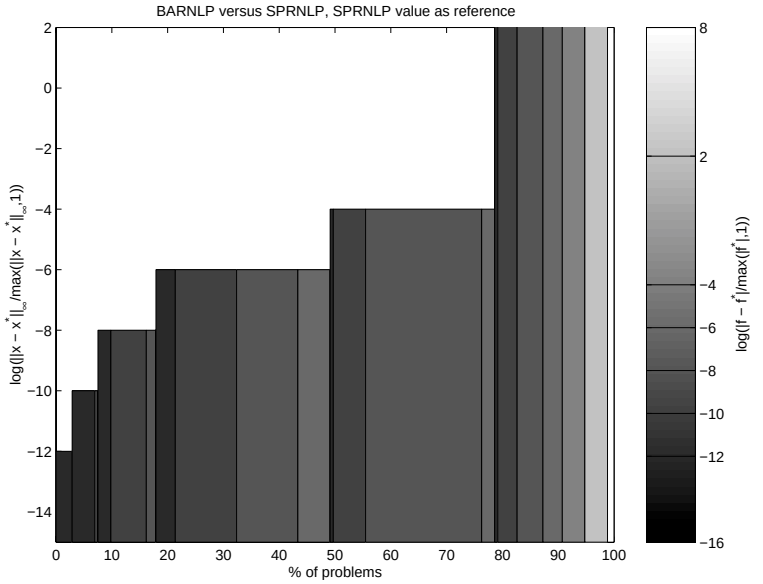


Figure 6: Comparison between BARNLP and SPRNLP, focus on the solution time.

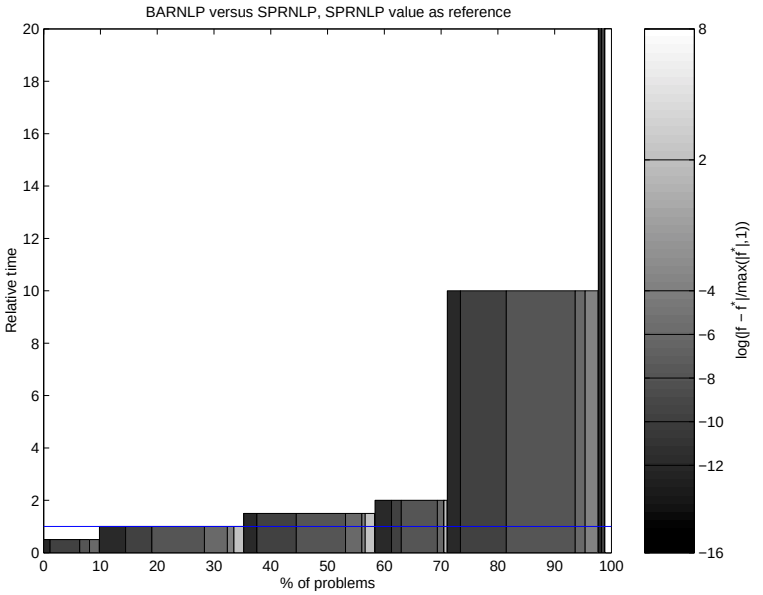


Figure 7: Comparison between SNOPT and SPRNLP, focus on objective function value.

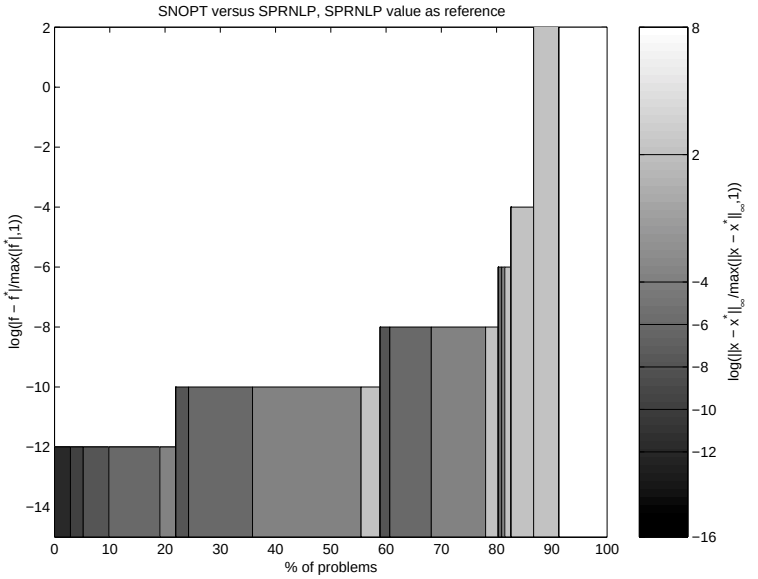


Figure 8: Comparison between SNOPT and SPRNLP, focus on coordinates of the solution.

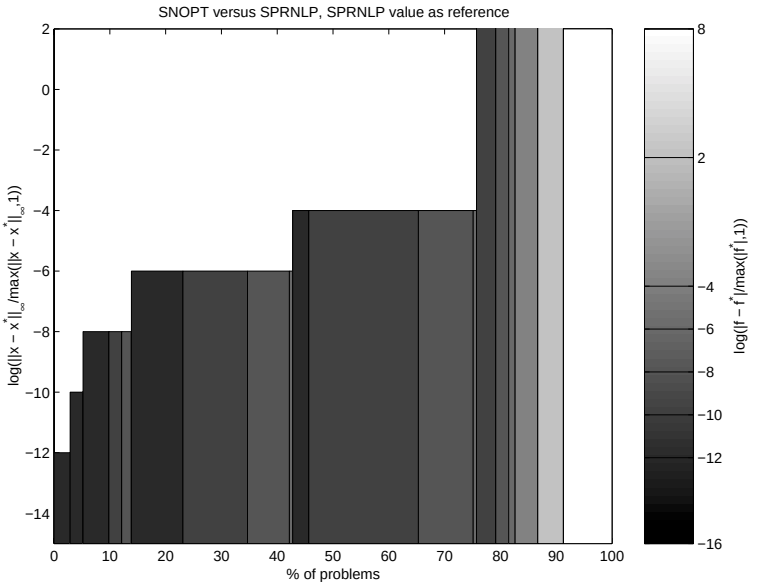
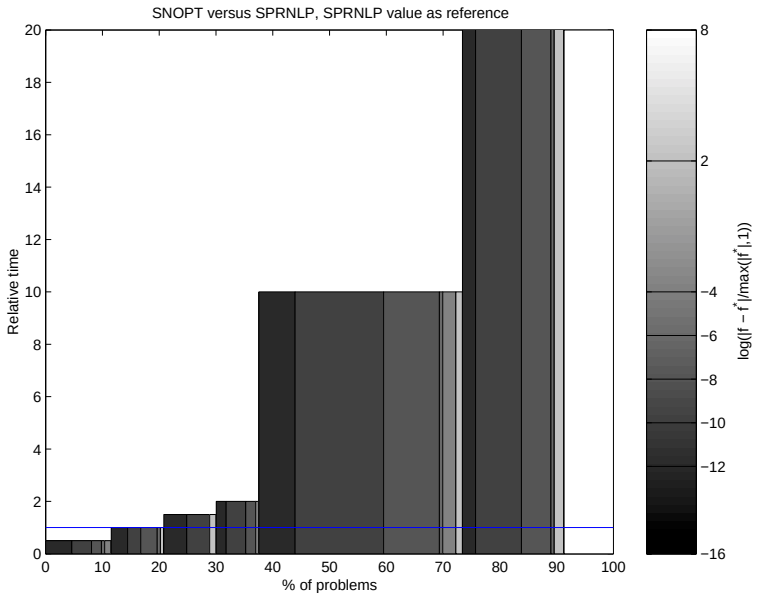


Figure 9: Comparison between SNOPT and SPRNLP, focus on the solution time.



and does not exploit Hessian sparsity. Therefore it is not surprising that the run time for many problems are considerably larger, and only for about 10 % of the problems does SNOPT demonstrate a run time advantage. The strength of SNOPT lies in its dependability.

4.3 Results for classes of problems

To look at the results in more detail, we divided the problems up into classes, and report how the different methods performed on these classes. The first classification is based on the size of the problem, whereas the other classification is based on the difficulty for an SQP method to solve a problem.

4.3.1 Classification by size

The most obvious classification of problems is by their size. We grouped the problems into five size classes (S1 to S5) as shown in Table 5. Here N is the number of unknowns, that is, the problem size. We show the performance of the four methods in Tables 6 through 10. Table 6 shows that the time differences between the four methods for small problems are small. SNOPT is the fastest method for this class of problems. The overhead for using second order information for the two Boeing Methods and LOQO shows clearly. When it finds a solution, LOQO is faster than BARNLP for the subset of problems in classes S2, S3 and S4. Unfortunately this performance advantage is negated by LOQO's

Class	Size	Number of Problems
S1	$0 < N \leq 100$,	51
S2	$100 < N \leq 500$,	50
S3	$500 < N \leq 1000$,	39
S4	$1000 < N \leq 2000$,	26
S5	$2000 < N$,	7

Table 5: Classification of problems by size.

Method	Converged	Warning	Local sol.	Failed	Total time	Time to solve 43 problems all methods could solve
SPRNLP	50	1	0	0	50	46
BARNLP	46	3	1	1	48	34
SNOPT	45	4	0	2	36	33
LOQO	43	0	3	5	144	47

Table 6: Performance of the methods for problems of class S1 ($0 < N \leq 100$).

inability to find a solution for many of the problems in these classes. SNOPT needs more time than all other methods as soon as the size of the problems becomes large, that is for size classes S2 through S5.For these problems it becomes very expensive not to use sparsity information of the Hessian. Nevertheless, if given enough time, SNOPT is able to solve nearly every problem.

4.3.2 Classification by computational complexity

Classification by size does not take into account the computational complexity of the problems. In particular, for problems with many inequality constraints, one might expect a performance degradation caused by the computational complexity of an active set algorithm such as the SQP codes SNOPT and SPRNLP.

Method	Converged	Warning	Local sol.	Failed	Total time	Time to solve 27 problems all methods could solve
SPRNLP	46	0	4	0	342	163
BARNLP	48	0	2	0	562	362
SNOPT	43	4	2	1	2497	1710
LOQO	29	0	1	20	6070	311

Table 7: Performance of the methods for problems of class S2 ($100 < N \leq 500$).

Method	Converged	Warning	Local sol.	Failed	Total time	Time to solve 15 problems all methods could solve
SPRNLP	37	0	2	0	894	265
BARNLP	39	0	0	0	1912	782
SNOPT	29	4	4	2	12694	5131
LOQO	17	0	0	22	19842	583

Table 8: Performance of the methods for problems of class S3 ($500 < N \leq 1000$).

Method	Converged	Warning	Local sol.	Failed	Total time	Time to solve 8 problems all methods could solve
SPRNLP	24	0	2	0	3227	213
BARNLP	22	1	2	1	7374	407
SNOPT	16	3	0	7	197417	80908
LOQO	10	0	0	16	33550	354

Table 9: Performance of the methods for problems of class S4 ($1000 < N \leq 2000$).

Method	Converged	Warning	Local sol.	Failed	Total time	Time to solve 4 problems all methods could solve
SPRNLP	7	0	0	0	1758	1241
BARNLP	7	0	0	0	2650	711
SNOPT	4	1	0	2	296684	1290643
LOQO	5	1	0	1	5906	2981

Table 10: Performance of the methods for problems of class S5 ($2000 < N$).

Class	Complexity	Number of Problems
H1	$1 \leq h_{SQP} \leq 3$,	23
H2	$3 < h_{SQP} \leq 5$,	60
H3	$5 < h_{SQP} \leq 10$,	52
H4	$10 < h_{SQP} \leq 20$,	25
H5	$20 < h_{SQP}$,	13

Table 11: Classification of problems by computational complexity.

Method	Converged	Warning	Local sol.	Failed	Total time	Time to solve 13 problems all methods could solve
SPRNLP	23	0	0	0	497	70
BARNLP	22	1	0	0	2333	325
SNOPT	18	2	1	2	8700	3446
LOQO	15	0	1	7	3174	360

Table 12: Performance of the methods for problems of complexity class H1.

This should be particularly apparent when a comparison is made with an interior point algorithm. Therefore we introduce another classification of the problems, which is based on the following observation: a problem is hard for an SQP method if the active set changes a lot from one nonlinear iteration to the next. We can measure this by looking at the number of QP iterations q and the number of gradient calls n_g. A measurement of how hard a problem is for an SQP method is then given by

$$h_{SQP} = \frac{q}{n_g}. \tag{6}$$

In Table 11 we show the five different classes we used when we classified the problems by their computational complexity for SQP methods. Problems in class H1 are "easiest", and the "hardest" problems are in class H5. Tables 12 through 16 show the performance of the four methods for all five classes.

Tables 12 through 15 show that as long as the number of active set changes are modest, the Boeing SQP method SPRNLP performs better, sometimes significantly, than all other methods. But when a large number of active set changes are needed, as for problems in H5, the Boeing Interior point method BARNLP is much faster. Furthermore it only gets trapped in one local minima, compared to three for SPRNLP. Since this categorization ignores problemsize (i.e. easy problems in H1 are both small and large), SNOPT's runtime in all classes is large, and the effect of the complexity classes is not apparent. LOQO performs well on the problems in H5, but again the algorithm reliability is apparent.

The measure h_{SQP} seems to be a good indicator to gauge the computational

Method	Converged	Warning	Local sol.	Failed	Total time	Time to solve 36 problems all methods could solve
SPRNLP	58	0	2	0	953	527
BARNLP	58	2	0	0	1979	1109
SNOPT	53	4	1	2	1289917	1286919
LOQO	38	0	0	22	14690	1133

Table 13: Performance of the methods for problems of complexity class H2.

Method	Converged	Warning	Local sol.	Failed	Total time	Time to solve 27 problems all methods could solve
SPRNLP	49	1	2	0	954	44
BARNLP	47	1	3	1	1292	78
SNOPT	40	6	1	5	24064	477
LOQO	30	0	2	20	15617	82

Table 14: Performance of the methods for problems of complexity class H3.

Method	Converged	Warning	Local sol.	Failed	Total time	Time to solve 14 problems all methods could solve
SPRNLP	24	0	1	0	2093	129
BARNLP	23	0	1	1	5979	299
SNOPT	16	4	1	4	75437	2823
LOQO	14	0	1	10	22611	235

Table 15: Performance of the methods for problems of complexity class H4.

Method	Converged	Warning	Local sol.	Failed	Total time	Time to solve 7 problems all methods could solve
SPRNLP	10	0	3	0	1773	1158
BARNLP	12	0	1	0	963	486
SNOPT	10	0	2	1	111210	84761
LOQO	7	1	0	5	9420	2466

Table 16: Performance of the methods for problems of complexity class H5.

complexity of NLP's for SQP methods. Further investigation is needed to find a way to automatically choose whether to use a SQP or interior point method.

We refer to appendix A for detailed results for all 173 problems.

5 Conclusions

We compared four different methods to solve NLP's. Two of these were SQP methods, and two were interior point methods. Overall the two Boeing implementations proved to be both most efficient and most successful. In a direct comparison between the two Boeing methods, overall the SQP method has an advantage, but for certain classes of problems the interior point method performs better. SNOPT is a very dependable method, since it can solve a large percentage of the test problems. However since it is a quasi-Newton method, and especially because it does not exploit Hessian sparsity, it needs much more time to solve the problems. This becomes very apparent once the problems grow in size. Finally, LOQO also efficiently solves the problems which are hard for the SQP methods, but overall is not as dependable as the other implementations.

6 Acknowledgments

The authors are grateful to C.T. Kelley, R.J. Vanderbei and Paul D. Frank for their help in preparing this report and their comments.

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A Appendix: Detailed results for all 173 test problems

In the following pages we give detailed results for all four methods on the 173 test problems. We used three different discretizations, H stands for Hermite Cubic

SPRNLP and BARNLP

IERNLP	Explanation
-133	Rank deficient Jacobian detected on successive iterations.
101	Weak solution found.
106	Maximum number of iterations in optimization phase.
107	Maximum number of iterations in feasibility phase.
108	Feasible point not found.
110	QP algorithm terminated.

SNOPT

IERNLP	Explanation
-160	All other errors.
104	Maximum number of function evaluations.
105	Feasible solution, but the requested accuracy in the dual infeasibilities could not be achieved.
108	Feasible point not found.
116	The current point cannot be improved.

LOQO

IERNLP	Explanation
-4242	Problem with the interface.
-1005	Primal and/or dual infeasible.
-1004	Dual infeasible.
-1002	Iteration limit reached.

Table 17: Error codes IERNLP for the different methods.

method, T stands for Trapezoidal, and R stands for explicit Runge-Kutta. N is the dimension of the problem, and M is the number of nonlinear constraints.

For all four methods we report the return code as IERNLP, the order of the constraint error as CONERR, the order of the boundary error as BDERR, and the order of the sum of these two errors. We also report the time each method spent on a problem. Table 17 explains the different error codes for the different methods.

For SPRNLP we also report the number of QP iterations, and the complexity factor h_{SQP}.

For the three other methods, we report the magnitude of the difference between the solutions for SPRNLP and the other methods, both absolute and relative as defined in equation 5. Finally we report the difference in the function value of the different solutions, and the order of the relative difference as defined in equation 4. Here f^* and x^* are the solution of SPRNLP.

Method	Case	1	2	3	4	5
	Problem	alpr01	aotv01	aqua01	arao01	arao02
	Ode	29	15	37	25	25
	Class	1	1	1	1	2
	Discr.	H	T	T	T	T
	N	166	176	140	50	75
	M	121	171	90	24	48
SPRNLP	IERNLP	0	0	0	0	0
	Total time	9.15	8.45	2.20	0.15	23.10
	CONERR	-10	-10	-12	-11	-10
	BDERR	-15	-15	-15	-15	-15
	SUM	-10	-10	-12	-11	-10
	QP Iterations	574	816	384	39	3963
	Factor	11.04	20.92	24.00	2.79	3.00
BARNLP	IERNLP	0	0	0	0	0
	Total time	13.64	7.32	0.73	0.16	0.09
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-11	-3	-10	-14	-10
	$f-f^*$	-2.41e-08	-1.85e-03	9.03e-09	4.55e-12	1.69e-10
	$\lVert x-x^*\rVert_\infty$	-4	0	-8	-7	-6
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-6	0	-9	-7	-6
	CONERR	-14	-10	-13	-13	-12
	BDERR	-15	-15	-15	-15	-15
	SUM	-14	-10	-13	-13	-12
SNOPT	IERNLP	105	105	0	105	0
	Total time	60.84	22.59	2.00	0.22	2.94
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-3	-2	-13	-14	-10
	$f-f^*$	-3.54e-01	1.79e-02	4.04e-12	5.46e-12	1.68e-10
	$\lVert x-x^*\rVert_\infty$	0	0	-6	-7	-6
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	0	0	-7	-7	-6
	CONERR	-12	-8	-12	-13	-12
	BDERR	-15	-15	-15	-15	-15
	SUM	-12	-8	-12	-13	-12
LOQO	IERNLP	0	-1002	0	0	0
	Total time	14.85	104.82	1.42	0.11	4.38
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-11	0	-10	-10	-9
	$f-f^*$	-2.75e-08	1.73e+00	-1.01e-08	-1.96e-07	-9.52e-10
	$\lVert x-x^*\rVert_\infty$	-4	0	-7	-7	-6
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-6	0	-8	-7	-6
	CONERR	-14	0	-10	-6	-9
	BDERR	-15	-2	-11	-13	-15
	SUM	-14	0	-10	-6	-9

Method	Case	6	7	8	9	10
	Problem	ashr01	ashr02	ashr03	ashr04	ashr05
	Ode	17	17	17	17	17
	Class	1	2	3	4	5
	Discr.	T	T	T	T	T
	N	20	20	20	20	80
	M	18	18	18	18	78
SPRNLP	IERNLP	0	0	0	0	0
	Total time	0.01	0.01	0.01	0.01	1.39
	CONERR	-14	-13	-14	-14	-8
	BDERR	-15	-15	-15	-15	-15
	SUM	-14	-13	-14	-14	-8
	QP Iterations	3	3	3	3	144
	Factor	3.00	3.00	3.00	3.00	3.69
BARNLP	IERNLP	0	0	0	0	101
	Total time	0.01	0.01	0.01	0.02	5.15
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-14	-14	-14	-14	-14
	$f-f^*$	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
	$\lVert x-x^*\rVert_\infty$	-15	-13	-14	-14	-1
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-14	-13	-14	-14	-1
	CONERR	-14	-14	-15	-15	-14
	BDERR	-15	-15	-15	-15	-15
	SUM	-14	-14	-15	-15	-14
SNOPT	IERNLP	0	0	0	0	108
	Total time	0.00	0.00	0.03	0.00	0.78
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-14	-14	-14	-14	-14
	$f-f^*$	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
	$\lVert x-x^*\rVert_\infty$	-12	-12	-11	-12	0
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-12	-12	-11	-12	0
	CONERR	-12	-12	-12	-12	-1
	BDERR	-15	-15	-15	-15	-15
	SUM	-12	-12	-12	-12	-1
LOQO	IERNLP	0	0	0	0	-1002
	Total time	0.02	0.03	0.02	0.02	19.36
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-14	-14	-14	-14	0
	$f-f^*$	0.00e+00	0.00e+00	0.00e+00	0.00e+00	1.91e+00
	$\lVert x-x^*\rVert_\infty$	-10	-7	-10	-12	0
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-10	-7	-10	-12	0
	CONERR	-9	-10	-10	-10	308
	BDERR	-11	-11	-11	-13	-4
	SUM	-9	-10	-10	-10	308

Method	Case	11	12	13	14	15
	Problem	bang01	brac01	brac02	brgr01	brn201
	Ode	22	10	10	5	28
	Class	1	1	2	1	1
	Discr.	T	T	T	T	T
	N	31	41	41	100	307
	M	18	27	37	98	257
SPRNLP	IERNLP	0	0	0	0	0
	Total time	0.14	0.35	0.36	1.02	22.65
	CONERR	-10	-8	-9	-14	-11
	BDERR	-15	-15	-15	-15	-15
	SUM	-10	-8	-9	-14	-11
	QP Iterations	58	137	140	121	570
	Factor	5.80	8.06	8.24	7.12	22.80
BARNLP	IERNLP	0	0	0	108	0
	Total time	0.10	0.85	0.39	5.20	14.16
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-7	-8	-8	-14	-11
	$f-f^*$	8.47e-08	1.05e-09	3.60e-09	0.00e+00	5.07e-12
	$\lVert x-x^*\rVert_\infty$	-6	-3	-3	0	-3
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-6	-3	-3	0	-3
	CONERR	-11	-8	-7	0	-11
	BDERR	-15	-15	-15	-15	-15
	SUM	-11	-8	-7	0	-11
SNOPT	IERNLP	0	0	105	108	0
	Total time	0.00	0.28	0.31	0.42	152.38
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-9	-8	-10	-14	-10
	$f-f^*$	-8.07e-10	1.04e-09	9.78e-11	0.00e+00	5.92e-11
	$\lVert x-x^*\rVert_\infty$	-8	-6	-5	0	-2
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-8	-6	-5	0	-2
	CONERR	-8	-12	-12	0	-12
	BDERR	-9	-15	-15	-15	-15
	SUM	-8	-12	-12	0	-12
LOQO	IERNLP	0	0	0	-1002	-1005
	Total time	0.04	0.11	0.20	22.99	517.50
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-8	-8	-10	-14	0
	$f-f^*$	1.05e-09	1.06e-09	8.71e-11	0.00e+00	1.38e+00
	$\lVert x-x^*\rVert_\infty$	-7	-3	-4	0	1
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-7	-3	-4	0	1
	CONERR	-9	-7	-10	0	8
	BDERR	-10	-13	-15	-3	-15
	SUM	-9	-7	-10	0	8

Method	Case	16	17	18	19	20
	Problem	brn202	brn203	brn204	capt01	capt02
	Ode	28	28	28	8	8
	Class	1	2	2	1	1
	Discr.	T	T	T	T	T
	N	307	271	271	587	587
	M	257	257	257	574	574
SPRNLP	IERNLP	0	0	0	0	0
	Total time	21.87	12.42	19.53	4.33	4.37
	CONERR	-9	-12	-10	-10	-9
	BDERR	-15	-15	-15	-15	-15
	SUM	-9	-12	-10	-10	-9
	QP Iterations	396	79	130	24	21
	Factor	18.00	4.39	5.65	2.67	2.33
BARNLP	IERNLP	0	0	0	0	0
	Total time	11.56	12.95	8.51	36.88	31.29
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-9	-10	-10	-9	-8
	$f-f^*$	7.23e-10	-3.20e-11	1.31e-11	1.35e-09	1.67e-09
	$\lVert x-x^*\rVert_\infty$	-2	-3	-3	-5	-5
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-2	-3	-3	-5	-5
	CONERR	-12	-10	-10	-9	-11
	BDERR	-15	-15	-15	-15	-15
	SUM	-12	-10	-10	-9	-11
SNOPT	IERNLP	0	0	0	0	0
	Total time	176.22	50.91	37.28	11.11	19.88
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-9	-10	-10	-10	-10
	$f-f^*$	7.36e-10	1.39e-11	5.04e-11	1.45e-11	8.64e-11
	$\lVert x-x^*\rVert_\infty$	-3	-2	-3	-4	-4
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-3	-2	-3	-4	-4
	CONERR	-12	-12	-12	-9	-10
	BDERR	-15	-15	-15	-15	-15
	SUM	-12	-12	-12	-9	-10
LOQO	IERNLP	-1004	-1004	-1004	-1002	-1002
	Total time	209.86	201.01	215.06	141.90	167.15
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	0	0	0	0	0
	$f-f^*$	1.38e+00	1.38e+00	1.38e+00	2.38e+00	2.38e+00
	$\lVert x-x^*\rVert_\infty$	1	1	1	0	0
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	1	1	1	0	0
	CONERR	8	15	15	-1	-1
	BDERR	-15	-15	-15	-1	-2
	SUM	8	15	15	-1	-1

Method	Case	21	22	23	24	25
	Problem	capt03	capt04	capt05	capt06	capt07
	Ode	8	8	8	8	8
	Class	2	2	3	3	4
	Discr.	T	T	T	T	T
	N	587	587	587	587	813
	M	574	574	574	574	877
SPRNLP	IERNLP	0	0	0	0	0
	Total time	7.00	7.12	4.33	4.37	31.96
	CONERR	-10	-9	-10	-9	-9
	BDERR	-15	-15	-15	-15	-15
	SUM	-10	-9	-10	-9	-9
	QP Iterations	32	30	24	21	90
	Factor	3.20	3.00	2.67	2.33	3.21
BARNLP	IERNLP	0	0	0	0	0
	Total time	33.92	31.25	34.20	31.40	94.92
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-8	-8	-9	-9	-5
	$f-f^*$	2.59e-09	3.24e-09	1.36e-09	1.74e-09	7.74e-06
	$\lVert x-x^*\rVert_\infty$	-5	-5	-5	-5	-1
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-5	-5	-5	-5	-1
	CONERR	-11	-10	-9	-12	-8
	BDERR	-15	-15	-15	-15	-15
	SUM	-11	-10	-9	-12	-8
SNOPT	IERNLP	0	0	0	105	0
	Total time	9.61	19.52	12.16	27.53	111.25
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-11	-10	-10	-10	-5
	$f-f^*$	3.02e-12	4.75e-11	4.14e-11	1.60e-10	-2.72e-06
	$\lVert x-x^*\rVert_\infty$	-4	-5	-5	-4	-1
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-4	-5	-5	-4	-1
	CONERR	-9	-11	-11	-10	-10
	BDERR	-15	-15	-15	-15	-15
	SUM	-9	-11	-11	-10	-10
LOQO	IERNLP	-1002	-1002	-1002	0	-1002
	Total time	139.44	139.95	226.36	27.35	171.49
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	0	0	0	-10	0
	$f-f^*$	1.70e+00	1.70e+00	-9.10e-01	1.43e-10	3.30e+00
	$\lVert x-x^*\rVert_\infty$	0	0	0	-5	0
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	0	0	0	-5	0
	CONERR	-1	-1	-1	-9	4
	BDERR	-2	-2	-2	-13	-2
	SUM	-1	-1	-1	-9	4

Method	Case	26	27	28	29	30
	Problem	capt08	capt09	capt10	capt11	capt12
	Ode	8	8	8	8	8
	Class	4	5	5	6	6
	Discr.	T	T	T	T	T
	N	813	1064	1064	1290	1290
	M	877	1063	1063	1366	1366
SPRNLP	IERNLP	0	0	0	0	0
	Total time	16.70	46.31	9.48	63.49	26.91
	CONERR	-11	-8	-10	-9	-11
	BDERR	-15	-15	-15	-15	-15
	SUM	-11	-8	-10	-9	-11
	QP Iterations	145	118	36	133	70
	Factor	8.53	4.21	3.60	3.91	4.12
BARNLP	IERNLP	0	0	0	0	0
	Total time	42.37	74.75	77.85	218.12	85.49
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-8	-7	-10	-5	-11
	$f-f^*$	1.60e-09	5.88e-08	1.13e-10	6.39e-06	1.20e-11
	$\lVert x-x^*\rVert_\infty$	-5	-1	-3	-1	-5
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-5	-1	-3	-1	-5
	CONERR	-10	-8	-8	-8	-10
	BDERR	-15	-15	-15	-15	-15
	SUM	-10	-8	-8	-8	-10
SNOPT	IERNLP	0	0	0	0	0
	Total time	51.58	86.44	60.84	305.97	143.20
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-11	-9	-10	-5	-10
	$f-f^*$	-6.90e-12	-1.06e-09	-2.35e-11	-2.71e-06	3.97e-11
	$\lVert x-x^*\rVert_\infty$	-4	-1	-4	-1	-3
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-4	-1	-4	-1	-3
	CONERR	-9	-10	-9	-9	-9
	BDERR	-15	-15	-15	-15	-15
	SUM	-9	-10	-9	-9	-9
LOQO	IERNLP	-1002	-1002	-1002	-1002	-1002
	Total time	183.65	258.97	257.94	312.18	307.37
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	0	0	0	0	0
	$f-f^*$	3.30e+00	3.26e+00	3.26e+00	3.26e+00	3.26e+00
	$\lVert x-x^*\rVert_\infty$	0	0	0	0	0
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	0	0	0	0	0
	CONERR	4	1	1	4	4
	BDERR	-2	-2	-1	-2	-2
	SUM	4	1	1	4	4

Method	Case	31	32	33	34	35
	Problem	capt13	capt14	chmr01	chmr02	chmr03
	Ode	8	8	20	20	20
	Class	7	8	1	2	3
	Discr.	T	T	T	T	T
	N	753	812	30	30	30
	M	802	876	18	18	18
SPRNLP	IERNLP	0	0	0	0	0
	Total time	49.03	8.73	0.16	0.20	0.19
	CONERR	-12	-9	-9	-10	-13
	BDERR	-15	-15	-15	-15	-15
	SUM	-12	-9	-9	-10	-13
	QP Iterations	523	36	62	90	93
	Factor	14.94	2.77	5.64	6.92	8.45
BARNLP	IERNLP	0	0	0	0	0
	Total time	48.91	41.70	0.12	0.13	0.18
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-4	-11	-8	-8	-7
	$f-f^*$	-3.59e-05	6.40e-12	2.40e-09	1.03e-09	1.00e-08
	$\lVert x-x^*\rVert_\infty$	0	-5	-6	-6	-7
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	0	-5	-6	-6	-7
	CONERR	-13	-10	-9	-10	-12
	BDERR	-15	-15	-15	-15	-15
	SUM	-13	-10	-9	-10	-12
SNOPT	IERNLP	105	0	0	0	0
	Total time	147.19	80.03	0.09	0.09	0.03
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-7	-11	-9	-10	-13
	$f-f^*$	-5.28e-08	7.73e-12	7.34e-10	4.46e-11	6.52e-14
	$\lVert x-x^*\rVert_\infty$	0	-4	-7	-6	-13
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	0	-4	-7	-6	-13
	CONERR	-11	-9	-12	-12	-15
	BDERR	-15	-15	-15	-15	-15
	SUM	-11	-9	-12	-12	-15
LOQO	IERNLP	-1002	-1002	0	0	0
	Total time	173.17	189.23	0.61	0.40	0.14
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	0	0	-9	-10	-10
	$f-f^*$	1.31e+00	3.29e+00	7.34e-10	4.55e-11	1.70e-11
	$\lVert x-x^*\rVert_\infty$	0	0	-6	-6	-9
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	0	0	-6	-6	-9
	CONERR	0	0	-15	-15	-10
	BDERR	-2	-2	-15	-15	-13
	SUM	0	0	-15	-15	-10

Method	Case	36	37	38	39	40
	Problem	chmr04	chmr05	chmr06	chmr07	chmr08
	Ode	20	20	20	20	20
	Class	4	5	6	7	8
	Discr.	T	T	T	T	T
	N	30	30	30	60	120
	M	18	18	18	38	78
SPRNLP	IERNLP	0	0	0	0	0
	Total time	0.15	0.19	0.14	0.44	0.84
	CONERR	-12	-9	-11	-11	-9
	BDERR	-15	-15	-15	-15	-15
	SUM	-12	-9	-11	-11	-9
	QP Iterations	70	86	53	106	144
	Factor	7.78	6.62	5.30	6.24	9.00
BARNLP	IERNLP	0	0	0	0	0
	Total time	0.10	0.12	0.14	0.25	0.60
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-7	-8	-8	-8	-8
	$f-f^*$	1.00e-08	7.61e-09	4.61e-09	1.04e-09	1.23e-09
	$\lVert x-x^*\rVert_\infty$	-7	-6	-3	-7	-7
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-7	-6	-3	-7	-7
	CONERR	-10	-10	-9	-11	-10
	BDERR	-15	-15	-15	-15	-15
	SUM	-10	-10	-9	-11	-10
SNOPT	IERNLP	0	0	0	0	0
	Total time	0.03	0.09	0.06	0.78	3.16
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-12	-9	-10	-10	-9
	$f-f^*$	2.43e-13	7.21e-10	1.84e-11	5.59e-11	3.51e-10
	$\lVert x-x^*\rVert_\infty$	-12	-6	-6	-9	-8
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-12	-6	-6	-9	-8
	CONERR	-15	-12	-12	-12	-12
	BDERR	-15	-15	-15	-15	-15
	SUM	-15	-12	-12	-12	-12
LOQO	IERNLP	0	0	-1005	0	0
	Total time	0.08	0.32	0.38	0.59	1.43
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-10	-9	-2	-10	-9
	$f-f^*$	3.19e-11	7.21e-10	5.41e-03	5.40e-11	3.44e-10
	$\lVert x-x^*\rVert_\infty$	-9	-6	-4	-8	-8
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-9	-6	-4	-8	-8
	CONERR	-11	-15	-15	-12	-12
	BDERR	-13	-15	-10	-10	-10
	SUM	-11	-15	-10	-10	-10

Method	Case	41	42	43	44	45
	Problem	chmr09	chmr10	clym01	clym02	clym03
	Ode	20	20	7	7	7
	Class	9	10	1	1	1
	Discr.	T	T	R	R	R
	N	150	30	180	450	900
	M	98	18	133	343	693
SPRNLP	IERNLP	0	0	0	0	0
	Total time	1.49	0.20	8.26	13.83	33.76
	CONERR	-9	-10	-10	-8	-10
	BDERR	-15	-15	-15	-15	-15
	SUM	-9	-10	-10	-8	-10
	QP Iterations	258	80	98	60	78
	Factor	15.18	5.33	4.26	3.75	4.33
BARNLP	IERNLP	0	0	0	0	0
	Total time	1.02	0.15	19.42	20.19	48.78
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-8	-8	-10	-8	-8
	$f-f^*$	3.52e-09	8.96e-09	1.57e-11	7.81e-09	2.73e-09
	$\lVert x-x^*\rVert_\infty$	-6	-5	-8	-6	-4
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-6	-5	-8	-6	-4
	CONERR	-10	-9	-11	-8	-8
	BDERR	-15	-15	-15	-15	-15
	SUM	-10	-9	-11	-8	-8
SNOPT	IERNLP	0	0	0	0	0
	Total time	4.50	0.06	15.28	53.69	169.84
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-8	-4	-10	-8	-11
	$f-f^*$	2.63e-09	-5.55e-05	1.50e-11	3.32e-09	5.96e-12
	$\lVert x-x^*\rVert_\infty$	-7	0	-6	-6	-5
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-7	0	-6	-6	-5
	CONERR	-12	-13	-12	-12	-12
	BDERR	-15	-15	-15	-15	-15
	SUM	-12	-13	-12	-12	-12
LOQO	IERNLP	-1002	0	0	0	-1002
	Total time	32.33	0.15	11.68	37.07	5251.89
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	0	-11	-9	-8	-4
	$f-f^*$	1.48e-01	8.04e-12	-3.56e-10	3.31e-09	-9.87e-05
	$\lVert x-x^*\rVert_\infty$	-7	-7	-8	-6	0
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-7	-7	-8	-6	0
	CONERR	0	-10	-9	-10	4
	BDERR	-15	-14	-11	-13	-2
	SUM	0	-10	-9	-10	4

Method	Case	46	47	48	49	50
	Problem	clym04	clym05	clym06	clym07	clym08
	Ode	7	7	7	7	7
	Class	1	1	1	1	1
	Discr.	T	T	T	H	H
	N	81	401	801	90	450
	M	63	343	693	63	343
SPRNLP	IERNLP	0	0	0	0	0
	Total time	0.85	4.34	10.67	2.73	15.54
	CONERR	-11	-11	-10	-8	-12
	BDERR	-15	-15	-15	-15	-15
	SUM	-11	-11	-10	-8	-12
	QP Iterations	56	58	81	57	61
	Factor	3.29	3.62	4.50	3.35	3.59
BARNLP	IERNLP	0	0	0	0	0
	Total time	1.44	5.84	16.06	4.97	22.70
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-12	-12	-9	-8	-8
	$f-f^*$	5.73e-13	1.36e-13	3.83e-10	1.92e-09	4.00e-09
	$\lVert x-x^*\rVert_\infty$	-10	-8	-5	-7	-6
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-10	-8	-5	-7	-6
	CONERR	-13	-12	-9	-11	-11
	BDERR	-15	-15	-15	-15	-15
	SUM	-13	-12	-9	-11	-11
SNOPT	IERNLP	0	0	0	0	0
	Total time	0.81	10.38	44.09	5.31	35.42
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-12	-12	-9	-8	-12
	$f-f^*$	5.62e-13	1.22e-13	2.13e-10	1.92e-09	1.03e-13
	$\lVert x-x^*\rVert_\infty$	-8	-6	-6	-7	-6
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-8	-6	-6	-7	-6
	CONERR	-11	-12	-12	-12	-12
	BDERR	-15	-15	-15	-15	-15
	SUM	-11	-12	-12	-12	-12
LOQO	IERNLP	0	-1002	0	0	0
	Total time	1.36	305.51	13.12	5.47	31.49
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-9	-1	-9	-8	-10
	$f-f^*$	-4.22e-10	5.06e-02	1.79e-10	1.49e-09	-1.63e-11
	$\lVert x-x^*\rVert_\infty$	-9	0	-6	-7	-8
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-9	0	-6	-7	-8
	CONERR	-9	4	-10	-9	-10
	BDERR	-12	-3	-13	-11	-12
	SUM	-9	4	-10	-9	-10

Method	Case	51	52	53	54	55
	Problem	clym09	clym10	clym11	clym12	clym13
	Ode	7	7	7	7	7
	Class	1	2	2	2	2
	Discr.	H	R	R	R	T
	N	900	140	350	700	61
	M	693	95	245	495	45
SPRNLP	IERNLP	0	0	0	0	0
	Total time	35.35	5.62	9.25	22.81	0.66
	CONERR	-11	-10	-8	-10	-11
	BDERR	-15	-15	-15	-15	-15
	SUM	-11	-10	-8	-10	-11
	QP Iterations	76	98	60	78	56
	Factor	4.22	4.26	3.75	4.33	3.29
BARNLP	IERNLP	0	0	0	0	0
	Total time	57.45	13.07	13.56	32.71	1.11
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-8	-10	-8	-8	-12
	$f-f^*$	6.00e-09	1.62e-11	7.81e-09	2.75e-09	5.92e-13
	$\lVert x-x^*\rVert_\infty$	-5	-8	-6	-4	-10
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-5	-8	-6	-4	-10
	CONERR	-10	-11	-8	-8	-13
	BDERR	-15	-15	-15	-15	-15
	SUM	-10	-11	-8	-8	-13
SNOPT	IERNLP	0	0	0	0	0
	Total time	137.39	11.83	41.91	138.34	0.73
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-11	-10	-8	-10	-12
	$f-f^*$	1.96e-12	1.56e-11	3.32e-09	1.03e-11	5.80e-13
	$\lVert x-x^*\rVert_\infty$	-5	-6	-6	-5	-8
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-5	-6	-6	-5	-8
	CONERR	-12	-12	-12	-12	-11
	BDERR	-15	-15	-15	-15	-15
	SUM	-12	-12	-12	-12	-11
LOQO	IERNLP	0	0	0	0	0
	Total time	64.46	7.50	23.15	41.36	0.76
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-9	-10	-8	-10	-9
	$f-f^*$	-1.29e-10	-2.44e-11	2.85e-09	-6.66e-11	-4.20e-10
	$\lVert x-x^*\rVert_\infty$	-6	-8	-6	-6	-9
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-6	-8	-6	-6	-9
	CONERR	-10	-10	-9	-10	-9
	BDERR	-13	-12	-12	-13	-11
	SUM	-10	-10	-9	-10	-9

Method	Case	56	57	58	59	60
	Problem	clym14	clym15	clym16	clym17	clym18
	Ode	7	7	7	7	7
	Class	2	2	2	2	2
	Discr.	T	T	H	H	H
	N	301	601	70	350	700
	M	245	495	45	245	495
SPRNLP	IERNLP	0	0	0	0	0
	Total time	3.16	7.67	1.62	8.85	19.83
	CONERR	-10	-10	-8	-11	-11
	BDERR	-15	-15	-15	-15	-15
	SUM	-10	-10	-8	-11	-11
	QP Iterations	58	81	57	61	76
	Factor	3.62	4.50	3.35	3.59	4.22
BARNLP	IERNLP	0	0	0	0	0
	Total time	4.37	17.14	3.23	12.84	34.25
	$\frac{\lvert f-f^* \rvert}{\lvert f^* \rvert}$	-10	-9	-8	-8	-8
	$f - f^*$	1.42e-11	2.66e-10	1.93e-09	3.99e-09	4.03e-09
	$\lVert x - x^* \rVert_\infty$	-8	-5	-7	-6	-4
	$\frac{\lVert x-x^* \rVert_\infty}{\lVert x^* \rVert_\infty}$	-8	-5	-7	-6	-4
	CONERR	-12	-9	-11	-11	-8
	BDERR	-15	-15	-15	-15	-15
	SUM	-12	-9	-11	-11	-8
SNOPT	IERNLP	0	0	0	0	0
	Total time	9.27	36.86	3.52	26.44	111.16
	$\frac{\lvert f-f^* \rvert}{\lvert f^* \rvert}$	-10	-9	-8	-11	-11
	$f - f^*$	1.42e-11	1.49e-10	1.93e-09	-7.30e-12	-9.69e-12
	$\lVert x - x^* \rVert_\infty$	-6	-6	-7	-6	-5
	$\frac{\lVert x-x^* \rVert_\infty}{\lVert x^* \rVert_\infty}$	-6	-6	-7	-6	-5
	CONERR	-11	-12	-12	-12	-12
	BDERR	-15	-15	-15	-15	-15
	SUM	-11	-12	-12	-12	-12
LOQO	IERNLP	-1002	0	0	0	0
	Total time	157.59	10.48	2.86	17.29	37.03
	$\frac{\lvert f-f^* \rvert}{\lvert f^* \rvert}$	-1	-10	-8	-10	-10
	$f - f^*$	5.06e-02	8.04e-11	1.32e-09	-7.04e-11	-4.92e-11
	$\lVert x - x^* \rVert_\infty$	0	-6	-7	-8	-7
	$\frac{\lVert x-x^* \rVert_\infty}{\lVert x^* \rVert_\infty}$	0	-6	-7	-8	-7
	CONERR	4	-10	-9	-10	-10
	BDERR	-2	-13	-11	-13	-13
	SUM	4	-10	-9	-10	-10

Method	Case	61	62	63	64	65
	Problem	clym19	clym20	dlay01	gdrd01	gdrd02
	Ode	7	7	33	4	4
	Class	3	4	1	1	1
	Discr.	T	T	T	R	T
	N	81	81	40	50	41
	M	63	63	38	27	27
SPRNLP	IERNLP	0	110	0	0	0
	Total time	2.39	2.92	0.05	0.22	0.19
	CONERR	-12	-8	-12	-14	-13
	BDERR	-15	-15	-15	-15	-15
	SUM	-12	-8	-12	-14	-13
	QP Iterations	136	142	9	72	54
	Factor	4.25	5.26	3.00	9.00	6.00
BARNLP	IERNLP	101	101	0	0	0
	Total time	3.71	3.27	0.04	0.25	0.19
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-12	-6	-14	-7	-7
	$f-f^*$	-2.84e-13	-1.05e-07	0.00e+00	1.90e-08	1.00e-08
	$\lVert x-x^*\rVert_\infty$	-6	-3	-11	-6	-7
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-6	-3	-12	-6	-7
	CONERR	-13	-13	-12	-13	-14
	BDERR	-15	-15	-15	-15	-15
	SUM	-13	-13	-12	-13	-14
SNOPT	IERNLP	0	0	0	0	0
	Total time	0.81	0.81	0.05	0.06	0.05
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-12	-6	-14	-10	-9
	$f-f^*$	-2.98e-13	-1.05e-07	0.00e+00	4.64e-11	2.11e-10
	$\lVert x-x^*\rVert_\infty$	-6	-3	-10	-9	-9
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-6	-3	-11	-9	-9
	CONERR	-11	-11	-11	-10	-9
	BDERR	-15	-15	-15	-15	-15
	SUM	-11	-11	-11	-10	-9
LOQO	IERNLP	0	0	0	0	0
	Total time	0.82	1.07	0.08	0.21	0.09
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-10	-6	-14	-9	-9
	$f-f^*$	-5.03e-11	-1.05e-07	0.00e+00	2.88e-10	7.28e-10
	$\lVert x-x^*\rVert_\infty$	-6	-3	-9	-8	-7
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-6	-3	-10	-8	-7
	CONERR	-10	-10	-10	-10	-9
	BDERR	-12	-13	-13	-11	-9
	SUM	-10	-10	-10	-10	-9

Method	Case	66	67	68	69	70
	Problem	gdrd03	gdrd04	gdrd05	gdrd06	gsoc01
	Ode	4	4	4	4	24
	Class	1	2	2	2	1
	Discr.	H	R	T	H	T
	N	50	250	201	250	825
	M	27	147	147	147	602
SPRNLP	IERNLP	0	0	0	0	0
	Total time	0.26	3.13	1.45	7.51	55.21
	CONERR	-14	-10	-9	-9	-12
	BDERR	-15	-15	-15	-15	-15
	SUM	-14	-10	-9	-9	-12
	QP Iterations	74	276	157	569	547
	Factor	9.25	19.71	12.08	21.88	10.94
BARNLP	IERNLP	0	0	0	0	0
	Total time	0.28	1.97	1.10	3.62	116.39
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-7	-7	-7	-7	-4
	$f-f^*$	1.90e-08	7.82e-08	4.02e-08	9.61e-08	-1.46e-05
	$\lVert x-x^*\rVert_\infty$	-6	-4	-5	-5	0
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-6	-5	-6	-5	0
	CONERR	-13	-13	-12	-10	-8
	BDERR	-15	-15	-15	-15	-15
	SUM	-13	-13	-12	-10	-8
SNOPT	IERNLP	0	0	0	0	105
	Total time	0.06	3.23	1.56	3.70	427.75
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-10	-10	-10	-4	-4
	$f-f^*$	5.21e-11	1.61e-10	2.31e-10	1.29e-04	7.08e-05
	$\lVert x-x^*\rVert_\infty$	-9	-4	-6	0	0
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-9	-5	-6	0	0
	CONERR	-10	-12	-12	-12	-10
	BDERR	-15	-15	-15	-15	-15
	SUM	-10	-12	-12	-12	-10
LOQO	IERNLP	0	0	0	0	-1002
	Total time	0.26	1.61	0.79	5.16	1086.19
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-9	-9	-9	-4	0
	$f-f^*$	3.91e-10	3.81e-10	-5.16e-10	-1.28e-04	-1.15e-01
	$\lVert x-x^*\rVert_\infty$	-8	-4	-6	0	0
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-8	-5	-6	0	0
	CONERR	-10	-9	-9	-9	0
	BDERR	-11	-10	-10	-12	-2
	SUM	-10	-9	-9	-9	0

Method	Case	71	72	73	74	75
	Problem	gydn01	gydn02	hang01	hang02	heat01
	Ode	9	9	14	14	23
	Class	1	2	1	1	1
	Discr.	T	T	T	H	T
	N	181	161	126	150	140
	M	153	134	96	96	119
SPRNLP	IERNLP	0	0	0	0	0
	Total time	2.63	2.03	1.17	2.01	0.30
	CONERR	-9	-11	-12	-11	-13
	BDERR	-15	-15	-15	-15	-15
	SUM	-9	-11	-12	-11	-13
	QP Iterations	87	75	95	134	29
	Factor	4.35	4.41	4.13	5.36	5.80
BARNLP	IERNLP	0	0	0	0	0
	Total time	4.15	2.64	1.24	1.33	0.50
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-8	-8	-3	-7	-8
	$f-f^*$	4.11e-09	4.13e-09	-1.13e-03	1.80e-08	8.09e-09
	$\lVert x-x^*\rVert_\infty$	-5	-5	0	-5	-4
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-5	-5	0	-5	-4
	CONERR	-10	-10	-11	-11	-13
	BDERR	-15	-15	-15	-15	-15
	SUM	-10	-10	-11	-11	-13
SNOPT	IERNLP	0	0	0	0	0
	Total time	4.97	4.36	1.72	3.72	0.38
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-10	-11	-3	-12	-14
	$f-f^*$	2.41e-11	-4.31e-12	-1.13e-03	-3.86e-13	7.60e-16
	$\lVert x-x^*\rVert_\infty$	-5	-6	0	-5	-7
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-5	-6	0	-5	-7
	CONERR	-11	-12	-11	-11	-11
	BDERR	-15	-15	-15	-15	-15
	SUM	-11	-12	-11	-11	-11
LOQO	IERNLP	0	-1004	0	0	0
	Total time	2.46	23.24	1.03	1.39	0.37
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-13	-10	-3	-3	-9
	$f-f^*$	9.67e-14	-3.11e-11	-1.13e-03	3.92e-04	7.84e-10
	$\lVert x-x^*\rVert_\infty$	-7	0	0	0	-4
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-7	0	0	0	-5
	CONERR	-9	0	-7	-6	-11
	BDERR	-14	-15	-14	-12	-13
	SUM	-9	0	-7	-6	-11

Method	Case	76	77	78	79	80
	Problem	heat02	imp201	imp202	imp203	imp204
	Ode	23	26	26	26	26
	Class	2	1	-1	2	-2
	Discr.	T	A	A	A	A
	N	2650	8	10	61	63
	M	2550	5	7	52	54
SPRNLP	IERNLP	0	0	0	0	0
	Total time	16.79	0.19	0.25	0.45	0.28
	CONERR	-9	-7	-11	-10	-9
	BDERR	-15	-15	-15	-15	-15
	SUM	-9	-7	-11	-10	-9
	QP Iterations	15	123	126	75	24
	Factor	2.50	12.30	11.45	6.82	2.67
BARNLP	IERNLP	0	0	0	0	0
	Total time	88.03	0.13	0.17	0.34	0.60
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-5	-9	-10	-11	-10
	$f-f^*$	-2.09e-06	2.21e-10	-3.17e-11	3.62e-12	2.10e-11
	$\lVert x-x^*\rVert_\infty$	0	-6	-7	-7	-8
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	0	-7	-7	-7	-8
	CONERR	-8	-9	-9	-11	-11
	BDERR	-15	-15	-15	-15	-15
	SUM	-8	-9	-9	-11	-11
SNOPT	IERNLP	0	0	0	0	0
	Total time	75.41	0.19	0.25	0.41	0.41
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-5	-9	-12	-10	-10
	$f-f^*$	-2.09e-06	2.33e-10	6.30e-13	-1.54e-11	2.12e-11
	$\lVert x-x^*\rVert_\infty$	0	-6	-9	-7	-8
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	0	-7	-9	-7	-9
	CONERR	-7	-12	-12	-12	-11
	BDERR	-15	-15	-15	-15	-15
	SUM	-7	-12	-12	-12	-11
LOQO	IERNLP	0	0	0	0	0
	Total time	157.97	0.27	0.35	6.01	8.89
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-5	-9	-11	-11	-10
	$f-f^*$	-2.09e-06	2.08e-10	-4.03e-12	3.38e-12	2.26e-11
	$\lVert x-x^*\rVert_\infty$	0	-6	-9	-7	-9
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	0	-7	-9	-7	-9
	CONERR	-10	-10	-9	-11	-10
	BDERR	-15	-15	-15	-15	-15
	SUM	-10	-10	-9	-11	-10

Method	Case	81	82	83	84	85
	Problem	jmp201	jmp202	kplr01	lnts02	lnts03
	Ode	27	27	18	2	2
	Class	1	2	1	2	2
	Discr.	T	T	T	R	R
	N	213	177	10	60	120
	M	168	168	10	36	76
SPRNLP	IERNLP	0	0	0	0	0
	Total time	4.67	4.98	0.03	0.34	0.88
	CONERR	-9	-10	-12	-7	-11
	BDERR	-15	-15	-15	-15	-15
	SUM	-9	-10	-12	-7	-11
	QP Iterations	54	65	18	76	134
	Factor	3.60	4.33	3.00	5.07	7.88
BARNLP	IERNLP	0	0	0	0	0
	Total time	7.65	5.95	0.03	0.25	0.39
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-8	-11	-14	-9	-10
	$f-f^*$	2.13e-09	5.24e-12	0.00e+00	9.39e-10	-1.06e-11
	$\lVert x-x^*\rVert_\infty$	-3	-4	-11	-4	-6
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-3	-5	-11	-5	-7
	CONERR	-11	-11	-11	-12	-9
	BDERR	-15	-15	-15	-15	-15
	SUM	-11	-11	-11	-12	-9
SNOPT	IERNLP	0	0	0	0	0
	Total time	27.03	9.56	0.00	1.60	6.90
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-8	-11	-14	-9	-14
	$f-f^*$	2.14e-09	-8.45e-12	0.00e+00	9.39e-10	2.89e-15
	$\lVert x-x^*\rVert_\infty$	-3	-5	-8	-4	-6
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-4	-5	-8	-5	-7
	CONERR	-11	-11	-8	-12	-12
	BDERR	-15	-15	-15	-15	-15
	SUM	-11	-11	-8	-12	-12
LOQO	IERNLP	-1002	-1002	0	0	0
	Total time	983.01	1118.15	0.02	1.26	1.66
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	0	0	-14	-9	-10
	$f-f^*$	2.51e+00	2.50e+00	0.00e+00	6.44e-10	-8.25e-11
	$\lVert x-x^*\rVert_\infty$	2	0	-9	-4	-8
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	1	0	-9	-5	-9
	CONERR	3	0	-9	-8	-9
	BDERR	-2	-4	-15	-15	-15
	SUM	3	0	-9	-8	-9

Method	Case	86	87	88	89	90
	Problem	lnts04	lnts05	lnts06	lnts07	lnts08
	Ode	2	2	2	2	2
	Class	2	2	2	2	2
	Discr.	R	T	T	T	T
	N	1200	51	101	1001	2001
	M	796	36	76	796	1596
SPRNLP	IERNLP	0	0	0	0	0
	Total time	40.56	0.29	0.64	11.45	39.43
	CONERR	-11	-8	-9	-12	-12
	BDERR	-15	-15	-15	-15	-15
	SUM	-11	-8	-9	-12	-12
	QP Iterations	1068	74	113	338	610
	Factor	62.82	3.89	5.65	19.88	35.88
BARNLP	IERNLP	0	0	0	0	0
	Total time	7.62	0.14	0.24	3.86	8.47
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-11	-9	-9	-11	-10
	$f-f^*$	1.55e-12	9.77e-10	1.98e-10	-2.83e-12	-5.31e-11
	$\lVert x-x^*\rVert_\infty$	-6	-4	-4	-6	-6
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-7	-5	-5	-7	-7
	CONERR	-12	-13	-11	-10	-8
	BDERR	-15	-15	-15	-15	-15
	SUM	-12	-13	-11	-10	-8
SNOPT	IERNLP	0	0	0	0	0
	Total time	383.22	0.48	1.82	215.88	989.85
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-12	-9	-9	-11	-10
	$f-f^*$	8.64e-13	9.77e-10	1.99e-10	1.73e-12	1.15e-11
	$\lVert x-x^*\rVert_\infty$	-5	-4	-4	-4	-4
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-6	-5	-5	-6	-5
	CONERR	-11	-12	-12	-12	-11
	BDERR	-15	-15	-15	-15	-15
	SUM	-11	-12	-12	-12	-11
LOQO	IERNLP	0	0	0	0	0
	Total time	9.04	0.22	2.77	2.72	6.51
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-9	-9	-11	-10	-9
	$f-f^*$	-2.62e-10	8.57e-10	-9.50e-12	4.75e-11	-2.91e-10
	$\lVert x-x^*\rVert_\infty$	-7	-4	-4	-7	-8
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-8	-5	-5	-8	-9
	CONERR	-9	-8	-8	-9	-9
	BDERR	-11	-14	-15	-11	-12
	SUM	-9	-8	-8	-9	-9

Method	Case	91	92	93	94	95
	Problem	lnts09	lnts10	lnts11	lnts12	lowt01
	Ode	2	2	2	2	19
	Class	2	2	2	2	1
	Discr.	T	H	H	H	T
	N	10001	60	120	1200	50
	M	7996	36	76	796	36
SPRNLP	IERNLP	0	0	0	0	0
	Total time	975.00	0.30	0.79	39.11	0.42
	CONERR	-12	-8	-11	-12	-9
	BDERR	-15	-15	-15	-15	-15
	SUM	-12	-8	-11	-12	-9
	QP Iterations	2868	76	134	1068	99
	Factor	168.71	5.07	7.88	62.82	5.50
BARNLP	IERNLP	0	0	0	0	0
	Total time	100.40	0.20	0.33	6.00	0.32
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-12	-9	-10	-12	-8
	$f-f^*$	-8.47e-13	9.06e-10	-1.06e-11	3.89e-13	2.71e-09
	$\lVert x-x^*\rVert_\infty$	-5	-4	-6	-6	-5
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-6	-5	-7	-7	-5
	CONERR	-10	-12	-9	-12	-15
	BDERR	-15	-15	-15	-15	-15
	SUM	-10	-12	-9	-12	-15
SNOPT	IERNLP	0	0	0	0	0
	Total time	82191.57	1.23	3.84	383.90	0.22
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-10	-9	-13	-12	-8
	$f-f^*$	6.03e-11	9.06e-10	1.27e-14	4.44e-13	2.71e-09
	$\lVert x-x^*\rVert_\infty$	-3	-4	-6	-5	-5
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-4	-5	-7	-6	-5
	CONERR	-9	-12	-11	-11	-12
	BDERR	-15	-15	-15	-15	-15
	SUM	-9	-12	-11	-11	-12
LOQO	IERNLP	-1002	0	0	0	-1002
	Total time	2331.73	0.23	1.26	5.46	14.39
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-5	-9	-9	-9	-1
	$f-f^*$	2.67e-06	5.21e-10	-6.96e-10	-2.85e-10	6.96e-02
	$\lVert x-x^*\rVert_\infty$	1	-4	-8	-8	0
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	0	-5	-9	-9	0
	CONERR	-15	-8	-8	-9	13
	BDERR	-7	-11	-14	-11	-6
	SUM	-7	-8	-8	-9	13

Method	Case	96	97	98	99	100
	Problem	lwbr01	mirv01	nlqr01	nlqr02	nlqr03
	Ode	36	30	6	6	6
	Class	1	1	1	2	2
	Discr.	T	T	T	T	T
	N	1176	1259	41	41	161
	M	1071	1155	30	45	180
SPRNLP	IERNLP	0	0	0	0	0
	Total time	147.04	26.41	0.78	0.31	2.88
	CONERR	-9	-9	-10	-9	-8
	BDERR	-15	-15	-15	-15	-15
	SUM	-9	-9	-10	-9	-8
	QP Iterations	1857	204	45	21	37
	Factor	16.01	9.27	3.21	2.62	3.36
BARNLP	IERNLP	106	0	0	0	0
	Total time	316.41	38.14	0.79	0.72	4.45
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-3	-7	-9	-12	-10
	$f-f^*$	-7.11e-04	-1.61e-08	-3.73e-10	8.93e-13	-1.10e-11
	$\lVert x-x^*\rVert_\infty$	0	-7	-7	-6	-6
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	0	-7	-7	-6	-6
	CONERR	-2	-11	-8	-9	-9
	BDERR	-15	-15	-15	-15	-15
	SUM	-2	-11	-8	-9	-9
SNOPT	IERNLP	104	0	0	0	0
	Total time	3408.76	3749.50	0.72	0.97	7.06
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-3	-7	-11	-12	-10
	$f-f^*$	1.69e-04	-1.61e-08	3.35e-12	-5.89e-13	-1.23e-11
	$\lVert x-x^*\rVert_\infty$	0	-4	-7	-6	-6
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	0	-4	-7	-6	-6
	CONERR	-5	-8	-9	-12	-11
	BDERR	-15	-15	-15	-15	-15
	SUM	-5	-8	-9	-12	-11
LOQO	IERNLP	-1005	-1005	0	0	0
	Total time	183.68	353.94	1.62	1.10	6.64
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	0	3	-10	-12	-12
	$f-f^*$	-1.21e+00	3.34e+03	-3.79e-11	-2.21e-13	1.23e-13
	$\lVert x-x^*\rVert_\infty$	0	0	-10	-6	-7
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	0	0	-10	-6	-7
	CONERR	1	1	-10	-10	-8
	BDERR	-15	-15	-15	-15	-15
	SUM	1	1	-10	-10	-8

Method	Case	101	102	103	104	105
	Problem	nlqr04	orbe02	orbe03	orbe04	orbe05
	Ode	6	13	13	13	13
	Class	2	1	1	1	2
	Discr.	T	T	T	T	T
	N	401	502	502	502	502
	M	450	397	397	397	397
SPRNLP	IERNLP	0	0	0	0	0
	Total time	15.13	18.56	55.66	12.49	36.94
	CONERR	-7	-7	-13	-11	-12
	BDERR	-15	-15	-15	-15	-15
	SUM	-7	-7	-13	-11	-12
	QP Iterations	66	327	283	116	797
	Factor	3.30	6.81	5.78	4.83	14.49
BARNLP	IERNLP	0	0	0	0	0
	Total time	14.96	24.36	107.33	11.22	13.04
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-5	-11	-13	-11	-11
	$f-f^*$	-1.71e-06	3.82e-12	5.98e-14	-3.36e-12	-6.61e-12
	$\lVert x-x^*\rVert_\infty$	-1	-7	-8	-6	-7
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-1	-7	-8	-6	-7
	CONERR	-9	-11	-12	-8	-9
	BDERR	-15	-15	-15	-15	-15
	SUM	-9	-11	-12	-8	-9
SNOPT	IERNLP	0	105	0	0	116
	Total time	25.86	600.92	4504.05	36.67	38.95
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-5	-11	-12	-12	0
	$f-f^*$	-1.71e-06	4.13e-12	-2.99e-13	-6.72e-13	1.84e-01
	$\lVert x-x^*\rVert_\infty$	-1	-5	-6	-6	0
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-1	-5	-6	-6	0
	CONERR	-12	-11	-12	-12	-8
	BDERR	-15	-15	-15	-15	-15
	SUM	-12	-11	-12	-12	-8
LOQO	IERNLP	0	-1002	-1002	-1005	-1002
	Total time	20.34	382.33	2494.64	77.08	363.89
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-5	0	0	0	0
	$f-f^*$	-1.71e-06	2.45e-01	2.44e-01	2.45e-01	2.04e-01
	$\lVert x-x^*\rVert_\infty$	-1	0	0	0	0
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-1	0	0	0	0
	CONERR	-8	0	0	0	0
	BDERR	-15	-6	-15	-15	-3
	SUM	-8	0	0	0	0

Method	Case	106	107	108	109	110
	Problem	orbe06	orbe07	orbe08	orbe09	orbe10
	Ode	13	13	13	13	13
	Class	2	2	3	3	3
	Discr.	T	T	T	T	T
	N	502	502	1002	1002	1002
	M	397	397	797	797	797
SPRNLP	IERNLP	0	0	0	0	0
	Total time	74.38	13.65	205.75	367.13	23.76
	CONERR	-11	-12	-8	-11	-8
	BDERR	-15	-15	-15	-15	-15
	SUM	-11	-12	-8	-11	-8
	QP Iterations	558	105	2422	2449	206
	Factor	9.79	3.89	21.43	22.47	8.58
BARNLP	IERNLP	0	0	0	0	0
	Total time	144.95	17.39	93.35	356.46	32.15
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-1	-3	-8	-1	-11
	$f-f^*$	-2.62e-02	-1.04e-04	7.57e-09	-1.85e-02	-2.57e-12
	$\lVert x-x^*\rVert_\infty$	0	0	-4	0	-5
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	0	0	-4	0	-5
	CONERR	-8	-10	-10	-12	-9
	BDERR	-15	-15	-15	-15	-15
	SUM	-8	-10	-10	-12	-9
SNOPT	IERNLP	116	105	104	0	105
	Total time	64.44	42.94	5912.52	20353.39	148.42
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	0	-12	-1	-1	-11
	$f-f^*$	1.36e-01	7.22e-13	-1.78e-02	-1.85e-02	6.59e-12
	$\lVert x-x^*\rVert_\infty$	0	-5	0	0	-5
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	0	-5	0	0	-5
	CONERR	-7	-11	-2	-12	-11
	BDERR	-15	-15	-15	-15	-15
	SUM	-7	-11	-2	-12	-11
LOQO	IERNLP	-1002	-1005	-1002	-1002	-1002
	Total time	3042.46	66.35	796.00	5418.24	633.48
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	0	0	0	0	0
	$f-f^*$	2.04e-01	2.30e-01	2.06e-01	2.05e-01	2.24e-01
	$\lVert x-x^*\rVert_\infty$	0	0	0	0	0
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	0	0	0	0	0
	CONERR	0	0	9	9	9
	BDERR	-3	-15	-4	-14	-4
	SUM	0	0	9	9	9

Method	Case	111	112	113	114	115
	Problem	orbe11	orbe12	orbe13	orbt01	orbt02
	Ode	13	13	13	12	12
	Class	4	4	4	1	2
	Discr.	T	T	T	T	T
	N	1502	1502	1502	1661	1661
	M	1197	1197	1197	1610	1610
SPRNLP	IERNLP	0	0	0	0	0
	Total time	465.15	932.63	111.12	64.30	148.34
	CONERR	-10	-9	-8	-10	-10
	BDERR	-15	-15	-15	-15	-15
	SUM	-10	-9	-8	-10	-10
	QP Iterations	3108	3208	386	63	258
	Factor	15.86	16.37	6.77	3.15	6.45
BARNLP	IERNLP	0	0	0	0	0
	Total time	262.95	1045.76	128.65	86.17	193.91
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-12	-11	-10	-8	-5
	$f-f^*$	3.44e-13	5.66e-12	-2.89e-11	-9.05e-09	-4.50e-06
	$\lVert x-x^*\rVert_\infty$	-5	-4	-7	0	1
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-5	-4	-7	-1	0
	CONERR	-11	-11	-8	-13	-10
	BDERR	-15	-15	-15	-15	-15
	SUM	-11	-11	-8	-13	-10
SNOPT	IERNLP	104	104	105	0	105
	Total time	16113.62	47607.67	4223.84	1741.78	6021.56
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-1	-2	-11	-8	-3
	$f-f^*$	1.39e-02	2.99e-03	1.65e-12	-8.93e-09	-2.07e-04
	$\lVert x-x^*\rVert_\infty$	0	0	-5	0	3
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	0	0	-5	-1	2
	CONERR	-3	-2	-9	-12	-12
	BDERR	-15	-15	-15	-15	-15
	SUM	-3	-2	-9	-12	-12
LOQO	IERNLP	-1002	-1002	-1002	-1005	-1005
	Total time	1104.13	6190.04	1098.12	2432.14	2755.48
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	0	0	0	0	0
	$f-f^*$	2.21e-01	2.20e-01	2.21e-01	5.68e-01	5.13e-01
	$\lVert x-x^*\rVert_\infty$	0	0	0	1	1
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	0	0	0	0	0
	CONERR	7	7	7	6	6
	BDERR	-3	-3	-3	-15	-15
	SUM	7	7	7	6	6

Method	Case	116	117	118	119	120
	Problem	orbt03	plnt01	pndl01	putt01	qlin01
	Ode	12	16	38	21	1
	Class	5	2	1	2	1
	Discr.	T	T	T	T	T
	N	1016	1470	60	61	90
	M	913	1418	46	68	54
SPRNLP	IERNLP	0	0	0	0	0
	Total time	90.96	198.16	1.19	2.21	0.11
	CONERR	-10	-8	-9	-12	-11
	BDERR	-15	-15	-15	-15	-15
	SUM	-10	-8	-9	-12	-11
	QP Iterations	270	301	217	185	22
	Factor	10.00	11.15	4.62	4.02	4.40
BARNLP	IERNLP	0	0	0	0	0
	Total time	124.69	3745.45	1.03	6.17	0.14
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-8	-11	-11	-7	-12
	$f-f^*$	6.77e-09	7.90e-12	8.10e-11	2.69e-08	-3.16e-13
	$\lVert x-x^*\rVert_\infty$	-5	-5	-9	-6	-10
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-5	-5	-9	-7	-11
	CONERR	-9	-9	-14	-12	-15
	BDERR	-15	-15	-15	-15	-15
	SUM	-9	-9	-14	-12	-15
SNOPT	IERNLP	104	116	0	105	105
	Total time	1931.98	4479.06	0.53	1.84	2.02
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	0	-3	-11	-10	-12
	$f-f^*$	-4.54e-01	-5.19e-04	1.00e-10	6.45e-11	-3.41e-13
	$\lVert x-x^*\rVert_\infty$	0	0	-6	-6	-8
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	0	0	-7	-7	-8
	CONERR	0	0	-12	-12	-12
	BDERR	-15	-15	-15	-15	-15
	SUM	0	0	-12	-12	-12
LOQO	IERNLP	-1004	-1002	0	0	0
	Total time	609.27	10375.80	1.16	2.98	0.16
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	0	0	-8	-11	-9
	$f-f^*$	3.28e-01	-3.44e-01	-1.91e-08	7.54e-12	-9.93e-10
	$\lVert x-x^*\rVert_\infty$	0	0	-8	-6	-8
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	0	0	-8	-7	-8
	CONERR	3	7	-8	-8	-8
	BDERR	-15	-4	-11	-15	-10
	SUM	3	7	-8	-8	-8

A Comparison of Interior Point and SQP Methods on Optimal Control Problems

Method	Case	121	122	123	124	125
	Problem	qlin02	qlin03	qlin04	robo01	robo02
	Ode	1	1	1	31	31
	Class	2	3	4	1	2
	Discr.	T	T	T	T	T
	N	100	90	10	100	90
	M	63	64	0	63	54
SPRNLP	IERNLP	0	0	0	0	0
	Total time	0.30	0.11	0.01	0.67	0.47
	CONERR	-9	-11	-15	-9	-12
	BDERR	-15	-15	-15	-15	-15
	SUM	-9	-11	-15	-9	-12
	QP Iterations	41	22	8	131	99
	Factor	3.42	4.40	2.67	10.92	11.00
BARNLP	IERNLP	0	0	0	0	0
	Total time	0.60	0.16	0.03	1.36	0.47
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-9	-12	-14	-8	-9
	$f-f^*$	9.16e-10	-3.16e-13	-7.35e-17	4.37e-09	3.97e-09
	$\lVert x-x^*\rVert_\infty$	-4	-10	-8	-5	-8
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-4	-11	-8	-5	-8
	CONERR	-11	-15	-15	-10	-15
	BDERR	-15	-15	-15	-15	-15
	SUM	-11	-15	-15	-10	-15
SNOPT	IERNLP	0	0	0	0	0
	Total time	2.01	2.12	0.00	0.62	0.72
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-9	-12	-14	-9	-12
	$f-f^*$	9.26e-10	-2.79e-13	-9.78e-17	3.24e-10	-1.69e-11
	$\lVert x-x^*\rVert_\infty$	-4	-7	-8	-5	-6
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-4	-7	-8	-5	-6
	CONERR	-12	-12	-15	-12	-12
	BDERR	-15	-15	-15	-15	-15
	SUM	-12	-12	-15	-12	-12
LOQO	IERNLP	0	0	-4242	0	0
	Total time	0.31	0.17	0.00	1.02	1.27
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-9	-9	0	-9	-10
	$f-f^*$	-3.01e-10	-7.65e-10	5.77e-01	-1.67e-10	-1.73e-09
	$\lVert x-x^*\rVert_\infty$	-4	-8	0	-5	-10
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-4	-8	0	-5	-10
	CONERR	-9	-9	-15	-10	-12
	BDERR	-11	-10	-15	-11	-12
	SUM	-9	-9	-15	-10	-11

Method	Case	126	127	128	129	130
	Problem	robo03	robo04	skwz01	skwz02	skwz03
	Ode	31	31	32	32	32
	Class	3	4	1	2	3
	Discr.	T	T	T	T	T
	N	91	287	1053	1092	1093
	M	54	277	1039	1039	1039
SPRNLP	IERNLP	0	0	0	0	0
	Total time	0.99	0.74	10.21	47.68	32.73
	CONERR	-8	-10	-10	-10	-9
	BDERR	-15	-15	-15	-15	-15
	SUM	-8	-10	-10	-10	-9
	QP Iterations	97	19	39	212	185
	Factor	6.06	4.75	3.00	7.57	10.88
BARNLP	IERNLP	0	0	101	0	0
	Total time	2.14	0.60	175.92	53.03	34.51
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-9	-14	-14	-1	-7
	$f-f^*$	5.52e-10	0.00e+00	0.00e+00	2.65e-02	3.90e-08
	$\lVert x-x^*\rVert_\infty$	-4	-11	-10	0	-4
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-4	-11	-10	0	-4
	CONERR	-8	-10	-11	-13	-10
	BDERR	-15	-15	-15	-15	-15
	SUM	-8	-10	-11	-13	-10
SNOPT	IERNLP	0	0	0	0	0
	Total time	1.09	1.09	18.35	87.92	45.70
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-11	-14	-14	-10	-13
	$f-f^*$	9.28e-12	0.00e+00	0.00e+00	-4.65e-11	-2.98e-14
	$\lVert x-x^*\rVert_\infty$	-4	-11	-9	-7	-9
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-4	-11	-9	-7	-9
	CONERR	-9	-12	-10	-11	-9
	BDERR	-15	-15	-15	-15	-15
	SUM	-9	-12	-10	-11	-9
LOQO	IERNLP	-1005	0	0	0	0
	Total time	39.85	2.37	126.02	67.50	50.66
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	1	-14	-14	-9	-9
	$f-f^*$	2.02e+01	0.00e+00	0.00e+00	4.51e-10	1.11e-10
	$\lVert x-x^*\rVert_\infty$	5	-9	0	0	0
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	5	-9	0	0	0
	CONERR	1	-9	-8	-10	-7
	BDERR	-15	-11	-12	-14	-11
	SUM	1	-9	-8	-10	-7

Method	Case	131	132	133	134	135
	Problem	skwz04	traj01	traj02	traj03	traj04
	Ode	32	3	3	3	3
	Class	4	1	1	1	1
	Discr.	T	R	R	T	T
	N	924	233	923	217	847
	M	877	185	755	185	755
SPRNLP	IERNLP	0	0	0	0	0
	Total time	20.67	10.58	47.81	3.36	17.66
	CONERR	-8	-12	-11	-12	-11
	BDERR	-15	-15	-15	-15	-15
	SUM	-8	-12	-11	-12	-11
	QP Iterations	143	132	303	113	150
	Factor	8.41	4.55	10.82	5.14	6.00
BARNLP	IERNLP	0	0	0	0	0
	Total time	16.14	14.05	104.75	3.40	29.50
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-9	-11	-10	-8	-11
	$f-f^*$	2.15e-10	-7.32e-12	-3.46e-11	1.00e-09	4.42e-12
	$\lVert x-x^*\rVert_\infty$	-7	-7	-7	-7	-10
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-8	-7	-7	-7	-10
	CONERR	-13	-11	-11	-12	-14
	BDERR	-15	-15	-15	-15	-15
	SUM	-13	-11	-11	-12	-14
SNOPT	IERNLP	0	0	0	0	0
	Total time	58.02	13.95	150.30	3.80	60.22
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-9	-12	-11	-12	-11
	$f-f^*$	2.08e-10	-4.66e-13	1.12e-12	-3.34e-13	4.35e-12
	$\lVert x-x^*\rVert_\infty$	-6	-6	-5	-5	-7
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-6	-6	-5	-5	-7
	CONERR	0	-12	-12	-11	-11
	BDERR	-15	-15	-15	-15	-15
	SUM	0	-12	-12	-11	-11
LOQO	IERNLP	0	-1002	-1002	-1002	-1002
	Total time	37.45	638.53	2642.78	134.61	605.82
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-9	0	0	0	0
	$f-f^*$	-6.59e-10	1.65e+00	1.65e+00	1.63e+00	1.65e+00
	$\lVert x-x^*\rVert_\infty$	0	2	1	1	1
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	0	2	1	1	1
	CONERR	0	13	-15	0	-15
	BDERR	-12	-1	-5	-2	-3
	SUM	0	13	-5	0	-3

Method	Case	136	137	138	139	140
	Problem	traj05	traj06	traj07	traj08	traj09
	Ode	3	3	3	3	3
	Class	1	1	2	2	2
	Discr.	H	H	R	R	T
	N	233	923	306	606	256
	M	185	755	200	400	200
SPRNLP	IERNLP	0	0	0	0	0
	Total time	9.82	50.14	8.83	39.96	2.65
	CONERR	-11	-10	-9	-11	-12
	BDERR	-15	-15	-15	-15	-15
	SUM	-11	-10	-9	-11	-12
	QP Iterations	129	222	109	244	80
	Factor	4.96	7.66	3.52	4.44	3.33
BARNLP	IERNLP	0	0	0	0	0
	Total time	10.90	50.70	12.39	20.28	3.24
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-8	-10	-9	-10	-11
	$f-f^*$	1.01e-09	3.33e-11	2.26e-10	1.07e-11	-3.66e-12
	$\lVert x-x^*\rVert_\infty$	-7	-9	-8	-9	-8
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-7	-9	-8	-9	-8
	CONERR	-12	-13	-11	-11	-13
	BDERR	-15	-15	-15	-15	-15
	SUM	-12	-13	-11	-11	-13
SNOPT	IERNLP	0	0	0	0	0
	Total time	12.43	168.73	30.77	83.16	6.77
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-11	-10	-9	-11	-10
	$f-f^*$	6.27e-12	3.33e-11	2.19e-10	3.33e-12	-1.30e-11
	$\lVert x-x^*\rVert_\infty$	-6	-6	-6	-5	-6
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-6	-6	-6	-5	-6
	CONERR	-12	-11	-11	-10	-11
	BDERR	-15	-15	-15	-15	-15
	SUM	-12	-11	-11	-10	-11
LOQO	IERNLP	-1002	-1002	-1002	-1002	-1005
	Total time	563.58	1508.49	75.61	128.57	40.23
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	0	0	0	0	0
	$f-f^*$	1.65e+00	1.65e+00	2.00e+00	2.00e+00	2.00e+00
	$\lVert x-x^*\rVert_\infty$	1	2	0	0	0
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	1	2	0	0	0
	CONERR	0	0	13	2	-5
	BDERR	-3	-2	-3	-3	-3
	SUM	0	0	13	2	-3

Method	Case	141	142	143	144	145
	Problem	traj10	traj11	traj12	traj13	traj14
	Ode	3	3	3	3	3
	Class	2	2	2	2	2
	Discr.	T	T	T	H	H
	N	506	1006	2006	306	606
	M	400	800	1600	200	400
SPRNLP	IERNLP	0	0	0	0	0
	Total time	11.42	30.18	74.44	4.92	11.03
	CONERR	-11	-11	-10	-12	-9
	BDERR	-15	-15	-15	-15	-15
	SUM	-11	-11	-10	-12	-9
	QP Iterations	180	235	314	88	100
	Factor	4.86	4.90	6.83	3.67	3.70
BARNLP	IERNLP	0	0	0	0	0
	Total time	10.25	18.51	54.26	10.49	14.85
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-12	-11	-9	-12	-11
	$f-f^*$	8.21e-13	-7.08e-12	-6.16e-10	2.10e-13	9.25e-12
	$\lVert x-x^*\rVert_\infty$	-9	-8	-6	-7	-6
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-9	-8	-6	-7	-6
	CONERR	-12	-11	-10	-12	-10
	BDERR	-15	-15	-15	-15	-15
	SUM	-12	-11	-10	-12	-10
SNOPT	IERNLP	0	0	0	0	116
	Total time	28.88	353.24	710.96	17.73	28.73
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-11	-10	-9	-12	0
	$f-f^*$	-5.80e-12	-2.19e-11	-4.94e-10	7.01e-13	-3.56e-01
	$\lVert x-x^*\rVert_\infty$	-5	-5	-4	-5	0
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-5	-5	-4	-5	0
	CONERR	-11	-11	-10	-11	0
	BDERR	-15	-15	-15	-15	-15
	SUM	-11	-11	-10	-11	0
LOQO	IERNLP	0	0	-1002	0	0
	Total time	30.90	15.27	994.14	11.15	26.59
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-9	-9	-4	-9	-8
	$f-f^*$	-2.15e-10	-7.93e-10	5.26e-05	-6.37e-10	-1.08e-09
	$\lVert x-x^*\rVert_\infty$	-9	-8	0	-8	-6
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-9	-8	0	-8	-6
	CONERR	-10	-9	4	-7	-9
	BDERR	-15	-12	0	-13	-15
	SUM	-10	-9	4	-7	-9

Method	Case	146	147	148	149	150
	Problem	traj15	traj16	traj17	traj18	traj19
	Ode	3	3	3	3	3
	Class	2	2	3	3	3
	Discr.	H	H	R	R	R
	N	1206	2406	458	458	908
	M	800	1600	250	250	500
SPRNLP	IERNLP	0	0	0	0	0
	Total time	38.91	59.35	7.20	7.42	16.20
	CONERR	-10	-10	-9	-9	-11
	BDERR	-15	-15	-15	-15	-15
	SUM	-10	-10	-9	-9	-11
	QP Iterations	185	172	47	108	54
	Factor	5.29	6.37	2.94	6.75	3.18
BARNLP	IERNLP	0	0	0	0	0
	Total time	33.84	94.67	27.89	24.29	41.02
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-11	-10	-11	-10	-11
	$f-f^*$	-6.15e-12	-1.69e-11	8.58e-12	-1.86e-11	8.53e-12
	$\lVert x-x^*\rVert_\infty$	-6	-8	-5	-5	-3
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-6	-8	-5	-5	-3
	CONERR	-10	-10	-8	-8	-9
	BDERR	-15	-15	-15	-15	-15
	SUM	-10	-10	-8	-8	-9
SNOPT	IERNLP	116	116	116	0	0
	Total time	132.34	897.94	13.29	127.24	1165.84
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	0	-1	-1	-11	-10
	$f-f^*$	-1.64e-01	-4.21e-02	-2.51e-02	-5.36e-12	-1.09e-11
	$\lVert x-x^*\rVert_\infty$	0	0	0	-4	-5
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	0	0	0	-4	-5
	CONERR	-1	-1	-1	-11	-12
	BDERR	-15	-15	-15	-15	-15
	SUM	-1	-1	-1	-11	-12
LOQO	IERNLP	0	0	-1002	-1002	0
	Total time	41.44	140.61	146.50	157.05	47.29
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-8	-9	0	0	-10
	$f-f^*$	-2.01e-09	-5.82e-10	-8.08e-01	-8.08e-01	-3.71e-11
	$\lVert x-x^*\rVert_\infty$	-8	-7	0	0	-4
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-8	-7	0	0	-4
	CONERR	-8	-8	13	13	-8
	BDERR	-12	-15	-3	-3	-14
	SUM	-8	-8	13	13	-8

Method	Case	151	152	153	154	155
	Problem	traj20	traj21	traj22	traj23	traj24
	Ode	3	3	3	3	3
	Class	3	3	3	3	3
	Discr.	R	T	T	T	T
	N	908	358	358	358	708
	M	500	250	250	250	500
SPRNLP	IERNLP	0	0	0	0	0
	Total time	18.83	3.54	3.67	12.79	6.93
	CONERR	-9	-9	-11	-9	-10
	BDERR	-15	-15	-15	-15	-15
	SUM	-9	-9	-11	-9	-10
	QP Iterations	182	61	111	334	53
	Factor	10.71	3.21	6.17	6.07	2.94
BARNLP	IERNLP	0	0	0	0	0
	Total time	36.50	7.06	14.42	13.73	15.07
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-7	-9	-10	-9	-10
	$f-f^*$	3.00e-08	6.30e-10	1.74e-11	3.58e-10	1.14e-11
	$\lVert x-x^*\rVert_\infty$	-3	-5	-6	-6	-4
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-3	-5	-6	-6	-4
	CONERR	-8	-9	-10	-10	-10
	BDERR	-15	-15	-15	-15	-15
	SUM	-8	-9	-10	-10	-10
SNOPT	IERNLP	0	105	105	0	0
	Total time	575.05	66.66	53.15	38.75	665.18
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-10	-9	-12	-9	-3
	$f-f^*$	1.51e-11	6.32e-10	-2.13e-13	3.28e-10	5.03e-04
	$\lVert x-x^*\rVert_\infty$	-3	-4	-4	-5	0
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-3	-4	-4	-5	0
	CONERR	-11	-11	-12	-12	-11
	BDERR	-15	-15	-15	-15	-15
	SUM	-11	-11	-12	-12	-11
LOQO	IERNLP	0	0	0	0	0
	Total time	60.33	5.12	5.82	18.68	12.80
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-10	-9	-10	-9	-11
	$f-f^*$	3.21e-11	6.15e-10	3.33e-11	3.31e-10	-3.85e-12
	$\lVert x-x^*\rVert_\infty$	-4	-5	-5	-7	-5
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-4	-5	-5	-7	-5
	CONERR	-8	-9	-9	-11	-8
	BDERR	-15	-13	-14	-15	-15
	SUM	-8	-9	-9	-11	-8

Method	Case	156	157	158	159	160
	Problem	traj25	traj26	traj27	traj28	traj29
	Ode	3	3	3	3	3
	Class	3	3	3	3	3
	Discr.	T	T	T	H	H
	N	708	1408	10011	458	458
	M	500	1000	7145	250	250
SPRNLP	IERNLP	0	0	0	0	0
	Total time	8.19	14.73	209.48	7.65	9.05
	CONERR	-8	-9	-8	-10	-8
	BDERR	-15	-15	-15	-15	-15
	SUM	-8	-9	-8	-10	-8
	QP Iterations	145	53	50	54	128
	Factor	8.53	2.94	3.12	3.18	7.11
BARNLP	IERNLP	0	0	0	0	0
	Total time	16.47	28.34	514.27	25.25	24.12
	$\frac{\lvert f-f^* \rvert}{\lvert f^* \rvert}$	-9	-9	-8	-10	-7
	$f - f^*$	-4.45e-10	2.83e-10	0.26e-08	1.91e-11	1.50e-08
	$\lVert x - x^* \rVert_\infty$	-4	-4	-4	-3	-3
	$\frac{\lVert x-x^* \rVert_\infty}{\lVert x^* \rVert_\infty}$	-4	-4	-4	-3	-3
	CONERR	-8	-9	-8	-9	-9
	BDERR	-15	-15	-15	-15	-15
	SUM	-8	-9	-8	-9	-9
SNOPT	IERNLP	0	0	105	0	0
	Total time	174.23	3320.56	1207386.34	146.81	188.72
	$\frac{\lvert f-f^* \rvert}{\lvert f^* \rvert}$	-10	-9	-9	-11	-10
	$f - f^*$	3.55e-11	2.79e-10	0.82e-09	9.24e-12	2.85e-11
	$\lVert x - x^* \rVert_\infty$	-4	-4	-2	-4	-4
	$\frac{\lVert x-x^* \rVert_\infty}{\lVert x^* \rVert_\infty}$	-4	-4	-2	-4	-4
	CONERR	-12	-12	-13	-11	-12
	BDERR	-15	-15	-15	-15	-15
	SUM	-12	-12	-13	-11	-12
LOQO	IERNLP	0	0	0	0	0
	Total time	11.98	34.30	484.97	23.96	26.55
	$\frac{\lvert f-f^* \rvert}{\lvert f^* \rvert}$	-8	-9	-8	-10	-10
	$f - f^*$	-2.64e-09	-2.00e-10	0.25e-08	-2.79e-11	2.11e-11
	$\lVert x - x^* \rVert_\infty$	-5	-4	-4	-5	-5
	$\frac{\lVert x-x^* \rVert_\infty}{\lVert x^* \rVert_\infty}$	-5	-4	-4	-5	-5
	CONERR	-8	-8	-9	-8	-9
	BDERR	-13	-14	-15	-14	-15
	SUM	-8	-8	-9	-8	-9

Method	Case	161	162	163	164	165
	Problem	traj30	traj31	traj32	traj33	traj34
	Ode	3	3	3	3	3
	Class	3	3	3	3	4
	Discr.	H	H	H	H	R
	N	908	908	1808	12869	458
	M	500	500	1000	7145	351
SPRNLP	IERNLP	0	0	0	0	0
	Total time	17.11	20.36	34.12	383.61	12.27
	CONERR	-9	-9	-9	-8	-12
	BDERR	-15	-15	-15	-15	-15
	SUM	-9	-9	-9	-8	-12
	QP Iterations	57	222	77	46	278
	Factor	3.17	12.33	4.53	2.88	15.44
BARNLP	IERNLP	0	0	0	0	0
	Total time	42.65	41.10	132.22	1789.76	50.96
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-9	-7	-9	-8	-10
	$f-f^*$	1.28e-10	3.00e-08	7.82e-10	5.06e-09	-4.84e-11
	$\lVert x-x^*\rVert_\infty$	-3	-3	-4	-3	-4
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-3	-3	-4	-3	-4
	CONERR	-9	-8	-9	-7	-8
	BDERR	-15	-15	-15	-15	-15
	SUM	-9	-8	-9	-7	-8
SNOPT	IERNLP	0	0	0	-160	104
	Total time	854.19	866.59	76186.88	4432.02	803.50
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-9	-11	-9	-3	-10
	$f-f^*$	1.24e-10	1.46e-12	7.83e-10	7.93e-04	3.06e-11
	$\lVert x-x^*\rVert_\infty$	-4	-4	-4	0	-4
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-4	-4	-4	0	-4
	CONERR	-12	-12	-12	-2	-12
	BDERR	-15	-15	-15	-15	-15
	SUM	-12	-12	-12	-2	-12
LOQO	IERNLP	0	0	0	0	0
	Total time	59.52	55.52	110.95	1789.92	22.30
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-10	-11	-9	-8	-9
	$f-f^*$	7.42e-11	6.73e-12	7.56e-10	4.91e-09	-1.23e-10
	$\lVert x-x^*\rVert_\infty$	-4	-5	-4	-4	-4
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-4	-5	-4	-4	-4
	CONERR	-8	-8	-8	-8	-8
	BDERR	-15	-15	-15	-15	-15
	SUM	-8	-8	-8	-8	-8

Method	Case	166	167	168	169	170
	Problem	traj35	traj36	traj37	traj38	traj39
	Ode	3	3	3	3	3
	Class	4	4	4	4	4
	Discr.	R	T	T	H	H
	N	908	358	708	458	908
	M	701	301	601	351	701
SPRNLP	IERNLP	0	0	0	0	0
	Total time	30.60	3.79	11.21	11.93	30.75
	CONERR	-8	-9	-9	-9	-8
	BDERR	-15	-15	-15	-15	-15
	SUM	-8	-9	-9	-9	-8
	QP Iterations	502	161	290	299	499
	Factor	27.89	11.50	17.06	17.59	27.72
BARNLP	IERNLP	0	0	0	0	0
	Total time	201.62	13.07	42.29	71.26	160.98
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-9	-9	-9	-9	-9
	$f-f^*$	3.59e-10	5.38e-10	3.29e-10	2.21e-10	9.75e-10
	$\lVert x-x^*\rVert_\infty$	-5	-5	-6	-5	-5
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-5	-5	-6	-5	-5
	CONERR	-9	-10	-9	-9	-8
	BDERR	-15	-15	-15	-15	-15
	SUM	-9	-10	-9	-9	-8
SNOPT	IERNLP	0	0	0	0	0
	Total time	398.81	42.38	161.00	105.84	411.34
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-9	-9	-9	-9	-9
	$f-f^*$	3.61e-10	5.89e-10	3.16e-10	2.20e-10	9.88e-10
	$\lVert x-x^*\rVert_\infty$	-4	-6	-4	-5	-3
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-4	-6	-4	-5	-3
	CONERR	-11	-12	-12	-12	-11
	BDERR	-15	-15	-15	-15	-15
	SUM	-11	-12	-12	-12	-11
LOQO	IERNLP	0	0	0	0	0
	Total time	59.86	5.28	11.18	21.28	52.15
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-9	-9	-9	-10	-9
	$f-f^*$	3.85e-10	-2.11e-10	2.50e-10	3.34e-11	9.92e-10
	$\lVert x-x^*\rVert_\infty$	-4	-4	-5	-4	-5
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-4	-4	-5	-4	-5
	CONERR	-8	-8	-8	-7	-8
	BDERR	-13	-13	-14	-13	-14
	SUM	-8	-8	-8	-7	-8

Method	Case	171	172	173
	Problem	tran01	wind01	zrml01
	Ode	39	11	40
	Class	1	1	1
	Discr.	T	T	T
	N	40	241	31
	M	18	235	18
SPRNLP	IERNLP	0	0	0
	Total time	0.26	4.30	0.17
	CONERR	-13	-9	-8
	BDERR	-15	-15	-15
	SUM	-13	-9	-8
	QP Iterations	90	480	113
	Factor	6.43	43.64	8.69
BARNLP	IERNLP	0	0	0
	Total time	0.21	2.10	0.07
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	0	-7	-7
	$f-f^*$	1.13e+00	1.80e-08	1.22e-08
	$\lVert x-x^*\rVert_\infty$	0	0	-6
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	0	0	-6
	CONERR	-14	-8	-10
	BDERR	-15	-15	-15
	SUM	-14	-8	-10
SNOPT	IERNLP	0	0	0
	Total time	0.17	4.34	0.17
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	-12	-10	-7
	$f-f^*$	8.23e-13	1.75e-11	1.23e-08
	$\lVert x-x^*\rVert_\infty$	-9	0	-7
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	-10	0	-7
	CONERR	-12	-12	-12
	BDERR	-15	-15	-15
	SUM	-12	-12	-12
LOQO	IERNLP	0	-1002	0
	Total time	0.20	112.20	0.05
	$\frac{\lvert f-f^*\rvert}{\lvert f^*\rvert}$	0	0	-7
	$f-f^*$	1.13e+00	4.90e-01	1.15e-08
	$\lVert x-x^*\rVert_\infty$	0	0	-7
	$\frac{\lVert x-x^*\rVert_\infty}{\lVert x^*\rVert_\infty}$	0	0	-7
	CONERR	-10	3	-9
	BDERR	-12	-3	-10
	SUM	-10	3	-9