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Model and Analysis of an Active Cradle System

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***Abstract* – One of the most challenging aspects of manufacturing Boeing products is the integration of large scale structures. In addition to the sheer size, airframes have specification tolerances that are stricter than structures which are much smaller. One specific example of this is the fuselage joining process. Not only must the segments be aligned, the gravity induced deformations must be accounted for. Recently, Boeing BR&T has been developing the so-called active cradle system to take on the challenges. In this paper, we describe the mathematical model used to predict the behavior of the fuselage sections as they are modified by the Cradle. In particular, we show that, subject to simple bounding conditions, it is not possible to effect local deformations that may damage the fuselage sections, using section 41 of the Boeing 787 as an example.**

***Index Terms* – fuselage join, active cradle, actuator, optimization, material damage, TaLLs.**

I. Introduction

Because of the massive scale involved, an aircraft fuselage is typically manufactured in sections. The sections are attached later by drilling holes and applying brackets and bolts. Since it is impossible to build the subsections to arbitrary precision, the fuselage sections may not align properly, and the attaching of sections must be delayed. There may be extensive additional work to overcome.

To address this challenge, Boeing is presently working on a so-called active or adaptive cradle. It consists of a frame with actuators positioned only along the lower half of the barrel, and forces can be applied to these actuators so that the barrel can be adequately deformed to match the contour of the adjacent section, allowing the fastening process to take place. Natural questions regarding such systems include (1) How does it operate and (2) What degree of risk, meaning unexpected damages to the fuselage, does it have? In this paper, we describe the mathematics associated with such a system and how to optimize the forces to achieve a desired displacement. More importantly, we provide conditions under which the system behavior leading to unexpected material damage can be controlled.

The basic notion of an active cradle, is that a section of the fuselage can sit in a cradle, and forces can be applied at different locations along the bottom half of the fuselage barrel edge to achieve desired displacements all around the barrel aligning the section for attachment with its adjacent fuselage section. Mathematical model and algorithms for computing the optimal forces needed to achieve desired displacement are fully described in a prior technical report [1]. One of the concerns related to such active control system is whether the process or applied forces can lead to an unexpected outcome in which the material is damaged. The purpose of this paper is to address such concern by establishing a theoretical result that provides conditions under which we can assure we are safe from causes of unexpected material damage. We further perform statistical analysis of the data obtained from actual experiments and derive statistical inferences.

The remainder of this report is organized as follows: Section II provides a brief review of the mathematical model for an active cradle system and the general optimization process for computing optimal forces to achieve a desired displacements; Section III discusses situations that can lead to unexpected outcomes in which the material is damaged. Theoretic result that establishes conditions under which we can assume we are safe from unexpected damage will be presented. Experimental results and statistical analyses will be presented in Section IV followed by a conclusion in Section V.

The following mathematical notations are used throughout this report:

- $x \in \mathbb{R}^n$: a vector of n actuator forces;
- $b \in \mathbb{R}^m$: a vector of m measured displacements from a pre-defined nominal shape;
- $\delta > 0$: the maximum allowable deviation from nominal shape;
- $L > 0$: the maximum allowable actuator force applied;
- $\varphi : \mathbb{R}^n \rightarrow \mathbb{R}^m$: the physical deformation of the barrel as an unknown function of applied forces;
- $A \in \mathbb{R}^{m \times n}$: the system control matrix used as an approximation of φ.

II. Mathematical Model And Active Control System For An Active Cradle

Given a barrel with initial displacements from nominal shape $b_0 \in \mathbb{R}^m$, the objective of the active cradle system is to compute forces $x \in \mathbb{R}^n$ so that the following condition is satisfied:

$$\|\varphi(x) + b_0\|_\infty \leq \delta, \tag{1}$$

where $\delta > 0$ is the maximum allowable deviation from nominal shape. That is, we seek to find forces x to deform the fuselage shape sufficiently to counteract the displacement b_0 so that all displacements from nominal are within the maximum allowable deviation.

We model the forces as being normal to the barrel with positive forces pointing toward the interior and negative forces pointing outward. The displacements are measured in normal-tangential coordinates, and we assume that m accounts for this; *i.e.*, there are actually $m/2$ measurement locations, each location with normal and tangential components, so that the resulting vector b is of size m. The system is currently designed to apply only normal forces. Our model and analysis still works when considering tangential forces, but the mathematics become a little more complicated (and the subsequent optimization a little more challenging).

Because we do not know exactly how the structure will behave in practice, the function φ is essentially a black box, which makes achieving the condition in (1) more challenging. In practice, each evaluation of φ at a given set of forces x can be accomplished either from a detailed finite element analyses (*i.e.*, simulations), or by actually physically deforming the barrel and taking precise measurements, e.g., by TaLLs system. The approach that we have taken here is to construct a system matrix A as an approximation of φ. This is done by choosing a set of suitably defined *unit* vectors of forces $\{x_i \in \mathbb{R}^n : i = 1, 2, \ldots, n\}$ from which we can construct any vector of forces as a linear combination of the unit vectors by applying each set of unit forces and measuring the resulting displacements. The resulting measurements become the columns of A. More specifically,

$$A = [\varphi(x_1), \varphi(x_2), \cdots, \varphi(x_n)]. \tag{2}$$

Mathematically, this set of force vectors is a basis for the vector space of possible force vectors. We typically use scalar multiples of the standard canonical basis ($x_i = e_i$, where e_i is the ith coordinate axis), but we are by no means restricted to it.

The condition in (1) means that the maximum deviation (or all deviations) from a desired shape must be less than δ. To make this problem more tractable, we can replace (1) with an optimization problem in which we replace $\varphi(x)$ with its approximation Ax, and minimize the forces required to achieve the desired displacements:

$$\min_{x \in R^n} \|x\|_p \quad \text{subject to} \quad \|Ax + b_0\|_\infty \leq \delta, \tag{3}$$

where p is any applicable norm. For simplicity, we prefer $p = 2$ or ∞.

Equivalently, we can place bounds on the allowable forces, and allow other norms in the problem. The resulting optimization problem is given as

$$\min_{x \in R^n} \|Ax + b_0\|_p \quad \text{subject to} \quad -L \leq x \leq L. \tag{4}$$

Efficiently solving any of these problems in practice requires a little more reformulation. Interested readers are referred to [1] for detailed treatment of the optimization procedure and other considerations such as actuator locations.

In practice, the optimization process formulated above can be applied iteratively to achieve a desired and optimal solution. An active control system iterates between applying optimal forces and updating the displacements as the process proceeds. In each iteration, two tasks are based on the behavior of the structure φ, and the optimization process O. For convenience, refer to the model formulation (4) for discussion.

As a function of the measured displacements b, the optimization process from (4) can be expressed as

$$x = O(b) = \arg \min_{-L \leq y \leq L} \|Ay + b\|_\infty .$$

We can represent the active control system as an iterative scheme, as follows:

$$x_{k+1} = O(b_k)$$
$$b_{k+1} = \varphi(x_{k+1}).$$

Together, this is a fixed point iteration

$$b_{k+1} = \varphi(O(b_k)) \equiv G(b_k),$$

where $G \equiv \varphi \circ O$ represents the composition of φ and O, or more explicitly,

$$b_{k+1} = \varphi\left(\arg \min_{-L \leq y \leq L} \|Ay + b_k\|_\infty\right) \tag{5}$$

Under reasonable assumptions, the existence of a solution and the convergence of the fixed-point iteration described above to the solution can be proved mathematically [1].

III. Conditions and Analysis of Material Damage

One of the major concerns related to the active control system is whether or not the optimization process (4) and the fixed-point iteration (5) can lead to an unexpected outcome in which the material is damaged. That is, unexpected displacements in the control loop which could damage the material. In this section, we establish conditions and theoretic result under which we can assume we are safe from this condition.

The possible causes of material damage include:

- Material strain limit unexpectedly exceeded;
- Inaccuracy in the system matrix approximation leads to unexpected wild behavior;
- Spike in displacement occurs in between adjacent measurement locations.

Material strain is computed at the Finite Element Analysis (FEA) level, and the strain limits are roughly 1.5% of the length of the element. Because this is assessed via FEA, strain checks cannot be efficiently performed inside a control loop. If we make the reasonable assumption that corrections to the initial optimal forces inside the control loop are small compared to the initial forces themselves, then the strains measured by the FEA when optimal forces are applied should be sufficient to enforce the safety concern that strain limits never be violated. It is unclear at this point how to incorporate strain limits into the optimization problem. Strain limit violation were never observed in earlier analysis.

The second and the third scenarios are actually related. We will show in the following theorem that when the realization of system matrix A via finite element analysis to the actual system behavior φ is accurate enough, *i.e.*, $\|Ax-\varphi(x)\|_\infty \le \varepsilon$, for some threshold $\varepsilon > 0$, then the displacement occurs in between adjacent measurement locations will not be radically different, *i.e.*, $|\varphi(x)_i-\varphi(x)_{i-1}| \le M,\ i=2,3,\dots,m,$ where M is a threshold condition above which damage would be expected to occur, m is the number of measurement locations, and x is a set of optimal forces described in previous sections. Here, we are assuming that we have enough measurements around that barrel to ensure that unexpected damage does not occur between measurement locations (*i.e.*, that all damage will be observed by measurements).

Theorem 1 *Let A be an approximation of the nonlinear structural behavior of the material* φ. *If*

$$\|Ax-\varphi(x)\| \le \varepsilon, \tag{6}$$

for some $\varepsilon > 0$ *and*

$$\|x\| \le \frac{c}{2\|A\|}, \tag{7}$$

for some $c > 0$, *then there exists* $M > 0$ *such that*

$$|\varphi(x)_i-\varphi(x)_{i-1}| \le M,\quad i=2,3,\dots,m, \tag{8}$$

where m is the number of measurement locations.

Proof. The proof is straightforward. For any given thresholds $\varepsilon > 0$ and $c > 0$, if we set $M \equiv 2\varepsilon + c$, then we have

$$\begin{aligned}
&|\varphi(x)_i-\varphi(x)_{i-1}| \\
&= |\varphi(x)_i-(Ax)_i+(Ax)_i-\varphi(x)_{i-1}-(Ax)_{i-1}+(Ax)_{i-1}| \\
&\le |\varphi(x)_i-(Ax)_i| + |(Ax)_{i-1}-\varphi(x)_{i-1}| + |(Ax)_i-(Ax)_{i-1}| \\
&= 2\varepsilon + |(a_i-a_{i-1})^T x| \\
&\le 2\varepsilon + \|a_i-a_{i-1}\|\|x\| \\
&\le 2\varepsilon + (\|a_i\|+\|a_{i-1}\|)\|x\| \\
&\le 2\varepsilon + 2\|A\|\|x\| \\
&\le 2\varepsilon + c \\
&\equiv M.
\end{aligned}$$

The importance of Theorem 1 is that it provides conditions under which no damage will occur. In practice, we are more interested in choosing the threshold parameters c and ε to ensure the Theorem holds.

Since the applied actuator forces x solve the optimization problem (4), it must satisfy the constraint with given maximum allowable force L. One natural choice for c is

$$c \equiv L\|A\|, \tag{9}$$

since this choice will ensure $\|x\| \le L$ which satisfies the constraint in the optimization model (4). In the next section, we will analyze data from experiments with fuselage section 41 and provide suggestions on parameter ε.

IV. Experimental Results and Statistical Analysis

Recall that the function φ is the physical deformation of the fuselage barrel as an unknown function of applied forces and is essentially a black box. The approach taken here is to construct a system matrix as an approximation of φ. In practice, the construction of a system control matrix can be accomplished either from a detailed FEA or by actually physically deforming the barrel and taking precise measurements. This is done by choosing a set of suitably defined *unit* vectors of forces, applying each set of unit forces, and measuring the resulting displacements. The resulting measurements become the columns of the system matrix.

Because of how fuselage sections are constructed, we only consider actuator positions that coincide with stringer locations on the bottom half of the barrel so that the structure is not damaged by applied forces. A possible actuator location (stringer) is called a *control node*. On the other hand, displacements are measured at specific locations around the fuselage barrel called response nodes. Control node and response node locations are given in terms of azimuth around the fuselage barrel, with the convention that the crown of the barrel is set at -180 degrees, increasing counterclockwise to 0 degrees at the keel and back to 180 degrees at the crown again.

Displacement at a specific measurement location i is given as simply $b_i = u_i - u_{0i}$, where u_i is the measurement position after forces are applied, u_{0i} is the location before any forces are applied, and the subtraction is component-wise. Finite element code and measurement systems typically generate different location data in *xyz* coordinates, where the x direction is along the length of the fuselage, y is across the barrel cross-section parallel to the ground, and z is across the height of the barrel. We typically ignore x since the displacements are so small relative to the other two coordinate directions. In addition, it is more convenient to convert from *yz* coordinate to normal-tangential coordinate because forces are applied only normal to the surface of the barrel.

The conversion to normal-tangential coordinates is done via the simple linear transformation $v = Ru$, where $u = (u_y, u_z)^T$ is a location in *yz* coordinates, $v = (v_n, v_t)^T$ is the same point in normal-tangential coordinates, and R is a rotation matrix defined by

$$R = \begin{bmatrix} N_z & N_y \\ -N_y & N_z \end{bmatrix}$$

with

$$N_y = -\sin\left(\frac{\pi}{2} - \tan^{-1}(u_y)\right)$$

and

$$N_z = -\cos\left(\frac{\pi}{2} - \tan^{-1}(u_z)\right).$$

To estimate the parameter ε in (6), we use the system matrix realized from TaLLs measurement, denoted by A, as our ground truth. Nonetheless these measurements reflect the real system deformations under applied forces. Then we compare A to the system matrix derived from finite element analysis, denoted by $\tilde{A}$.

A fuselage section 41 was used in our experiment. There are 10 control nodes, 241 response nodes from TaLLs and 71 from FEA. Although forces were only applied in the normal direction so that displacements in the tangential direction are small and can be ignored, displacements in both directions were recorded and analyzed. In addition, locations of response node results from FEA are smaller than those of TaLLs. A smooth spline model is used to interpolate the displacements simulated by FEA to the response locations taken by TaLLs so the differences can be computed.

To perform the analysis, we took advantage of the great software tool "*Joint Fuselage Analysis Matlab Tool*" developed by Mark Abramson [2] under the Boeing Active Cradle Project. Below are the steps we performed:

- Run Joint Fuselage Analysis Matlab Tool [2] on Section 41 Test by loading data from both FEA simulation and TaLLs measurements;
- Extract system matrices and response nodes;
- Construct a spline model for each column of the system matrix of FEA simulation and interpolate the model with the response nodes from TaLLs;
- Compute the component-wise residual

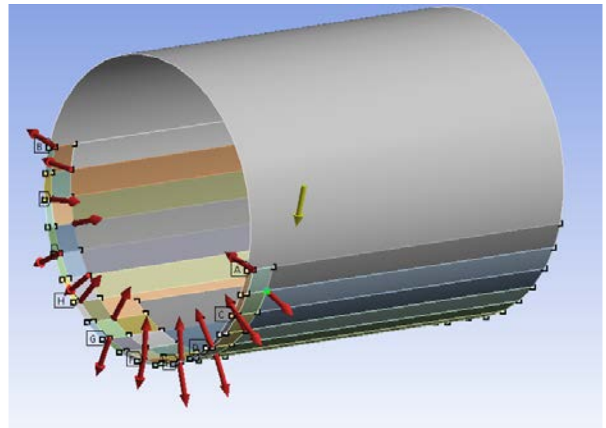


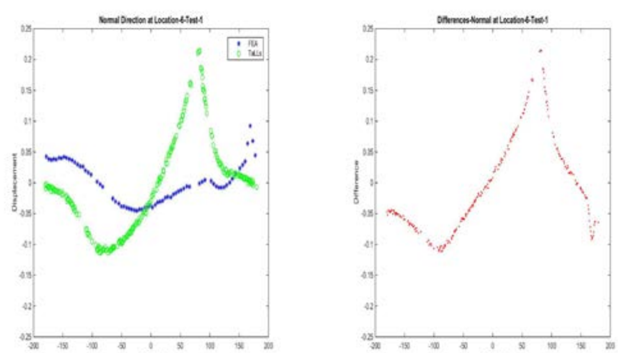
Figure 1: Schematic of Fuselage Barrel

A Model of the fuselage barrel is shown. The deformation forces are represented by arrows. In this particular case, forces are shown that are applied radially to the edge of the barrel. Tangential forces would be applied at the base of those arrows, perpendicular to the radial forces.

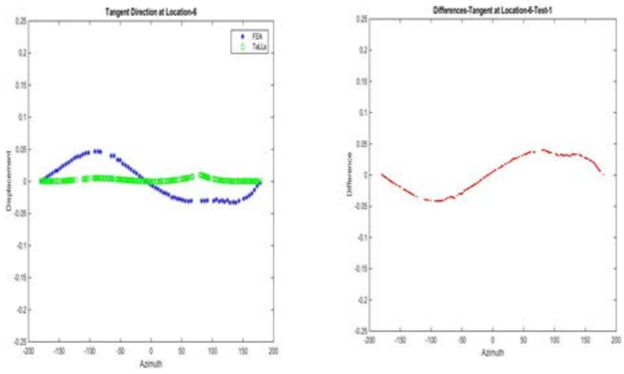
The maximum residual (the worst case scenario) we discovered is

$$\left|\tilde{A}_{ij} - A_{ij}\right| = 0.2143 \tag{10}$$

In Figure 2 we compare the displacements at response locations from FEA (blue curve) and TaLLs (green curve) and the differences between the simulation and the actual measurements when forces were applied at stringer location S38L. Comparisons in normal direction and tangential direction are made separately. Displacements in the tangential direction are in fact very small. Note that this is the worst case output in our comparison. Much smaller discrepancies were observed at all other locations.



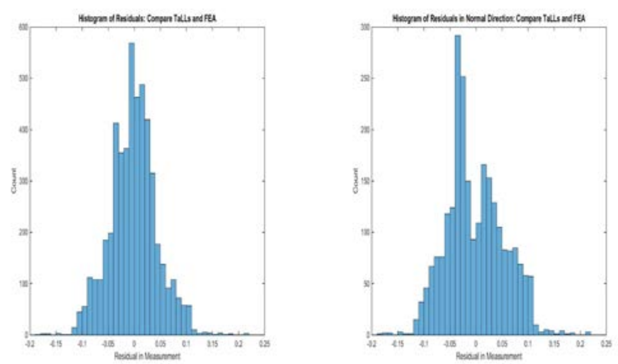
(a) (b)



(c) (d)

Figure 2: Comparison of displacements (mils) and residuals at location S38L. (a) displacements from FEA (blue) and TaLLs (green) in normal direction; (b) residuals in normal direction; (c) displacements from FEA (blue) and TaLLs (green) in tangential direction; (d) residuals in tangential direction.

We further preform statistical analysis on the residuals. Figure 3(a) shows the histogram of residuals in all directions. The distribution is close to normal distribution with mean -0.0010 and standard deviation 0.0449. If we focus only on the normal direction, the distribution is similar with mean -0.0016 and standard deviation 0.0552 and is shown in Figure 3(b). The box plot in Figure 4 shows that both distributions are indeed very similar with small interquartile range (IQR).



(a) (b)

Figure 3: Figures of (a) Histogram of all residuals; (b) Histogram of residuals in the normal direction.

At last, we estimate the probabilities of residuals worse than what we observed in (10). They are

$$p\left(\left|\tilde{A}_{ij} - A_{ij}\right| \geq 0.2143\right) = 1.6323 \times 10^{-6}$$

for both normal and tangential directions, and

$$p\left(\left|\tilde{A}_{ij} - A_{ij}\right| \geq 0.2143\right) = 90.504 \times 10^{-6}$$

for normal direction only. Both are indeed very small.

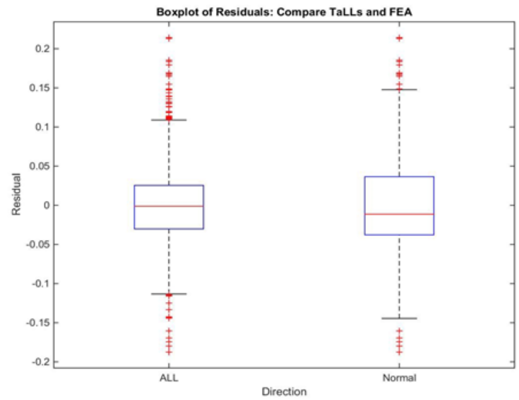


Figure 4: Box plots of residuals in both directions (left) and in the normal direction only (right).

V. Conclusion

In this report, we described the mathematical model for an active cradle system to achieve desired displacements all around the aircraft barrel to align the sections for attachment. We also describe an iterative control procedure for computing the optimal forces that can be applied at different actuator locations along the bottom half of the fuselage barrel. Most importantly, we established conditions under which no material damage will occur, and we reported the choice of threshold parameters that governs these conditions

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Dr. William Ferng, is a Senior Advanced Math Modeler who has a Ph.D. degree in Applied Mathematics from North Carolina State University. Ferng joined Boeing in 2001. Since then he has led or been a critical member of variety of projects. The wide range of applications he has worked on includes active cradle system for aircraft fuselage, micro-scale combustion calorimeter, statistical analysis, text mining, and social network analysis. He was Co-PI for government research contracts from DARPA (Defense Advanced Research Projects Agency) and ONR (Office of Naval Research). He has a strong background in mathematics, specializing in linear algebra and matrix computations. He has published in prestigious journals in areas ranging from numerical algorithms to parallel computing, served as a paper reviewer for journals and in program committees for international conferences.

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