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An Historical Perspective on Boeing's Influence on Dynamic Structural Analysis Numerical Simulation

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***Abstract* – Boeing is well known for the aerospace products it has produced over its 100-year history. What is not so well known is that Boeing played a major role in the development of the Finite Element Method, Lanczos Eigenvalue Extraction and Craig-Bampton Reduction. These three numerical simulation methods revolutionized large system structural dynamic analysis when brought together and deployed in commercial software packages. Boeing utilizes these methods as the foundations of aircraft aeroelastic loads and flutter analysis, propulsion structural dynamic analysis and space vehicle and satellite coupled loads analysis. Outside of Boeing, these methods are commonly applied in automotive vehicle dynamic and noise, vibration, and harshness analysis as well as a multitude of other industries producing consumer products and heavy machinery.**

***Index Terms* – Finite Element Analysis, Finite Element Method, Discrete Stiffness Method, Lanczos Eigenvalue Extraction, Craig-Bampton, Component Modal Syntheses**

***Acronyms* – Automated Component Modal Synthesis (ACMS), Automated Multi-Level Substructuring System (AMLS), Boeing Computer Services (BCS), Boeing Commercial Airplane (BCA), Craig-Bampton (C-B), Discrete Stiffness Method (DSM), Degree of Freedom (DOF), Engine Vibration Related Noise (EVRN), Fan Blade Out (FBO), Finite Element Analysis (FEA), Finite Element Method (FEM), Intellectual Property (IP), International Traffic in Arms Regulations (ITAR), NASA Structural Analysis (NASTRAN), Noise Vibration and Harshness (NVH), Output Transformation Matrix (OTM), Request For Proposal (RFP), Senior Technical Fellow (STF), Graphical User Interface (GUI).**

I. INTRODUCTION

As Boeing celebrates its centennial, it is appropriate to reflect upon Boeing's impact on the development and deployment of several numerical solution methods pervasive in the field of structural dynamics. In particular, the convergence of the Finite Element Method, Craig-Bampton Reduction and the Lanczos Eigenvalue extraction method formed a foundation that has improved and accelerated the development and performance of virtually every modern Boeing airplane along with a multitude of consumer products familiar to everyone. For example, consider the modern automobile. Today's automobiles have superb handling and are extremely quiet compared to vehicles 30 years ago. The comfortable environment of a modern automobile is largely due to the industry's focused efforts on Noise, Vibration, and Harshness (NVH) analysis as a means to improve their product and sales. At the heart of each NVH analysis is a multi-million DOF finite element vibro-acoustic model. The low to mid frequency acoustic and structural responses require that thousands of frequencies and mode shapes be calculated. Without some form of substructuring and dynamic reduction, runtimes are exorbitant. The Automated Multi-Level Substructuring (AMLS) method [20] and Automated Component Modal Synthesis (ACMS) method [21] are routinely applied to the NVH analyses yielding results in hours instead of days or weeks. At the core, these methods utilize component modal reduction and synthesis techniques linked back to the Craig-Bampton technique that requires an eigenvalue/ eigenvector calculation/ extraction on the substructure and this is typically done with the Lanczos method due to its proven accuracy and performance. In the following sections, we look into the role Boeing played in the inspiration, development, and deployment of these numerical solution methods and summarize how these methods are used both within Boeing and outside of Boeing in the development of multitudes of products.

II. BOEING AND THE FINITE ELEMENT METHOD

Today within Boeing, the finite element method is pervasive with several thousand engineers utilizing FEA on a regular basis. Concerning structural analysis, it is central to loads and dynamics analysis as well as stress analysis and testing. It turns out that it was Boeing's need and desire to improve flutter prediction that led to Boeing's leading role in the development of the finite element method.

Who invented finite elements? In the publication "The Origins of the Finite Element Method" [1], Carlos Felippa states:

> *"Not just one individual, as this historical sketch will make clear. But if the question is tweaked to: who created the FEM in everyday use? there is no question in the writer's mind: M. J. (Jon) Turner at Boeing over the period 1950–1962. He generalized and perfected the Direct Stiffness Method, and forcefully got Boeing to commit resources to it while other aerospace companies were mired in the Force Method. During 1952–53 he oversaw the development of the first continuum based finite elements."*

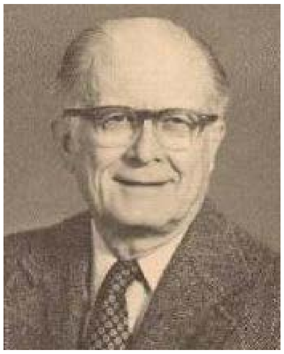
Figure 1: Jon Turner

Jon Turner was the supervisor of the Structural Dynamics Unit at Boeing in Seattle. In the early 1950's, with the growing popularity of jet aircraft, and with demands for high performance military aircraft, delta wing structures presented new modeling and analysis problems. Existing unidirectional (that is, beam models) models did not provide sufficient accuracy. Instead, two-dimensional panel elements of arbitrary geometry were needed.

At this time, Boeing had a summer faculty program, whereby faculty members from universities were invited to work at Boeing over the summer. In the summers of 1952-53, Jon Turner invited Ray Clough from the University of California at Berkley, and Harold Martin from the University of Washington to work for him on a method to calculate the vibration properties for the low-aspect ratio box beam. This collaboration resulted in the seminal paper by Turner, Clough, Martin and Topp in 1956 [2] which summarized a procedure called the Direct Stiffness Method (DSM) and derived a constant strain triangular element along with a rectangular membrane element. (Topp was a structures engineer at the Boeing Airplane Company, Wichita Division.)

It is apropos to hear this story in the words of Clough. The following passage is from a speech by Clough transcribed and published in 2004. [3]:

> *"When I applied for the Boeing Summer Faculty job in June 1952, I was assigned to the Structural Dynamics Unit under the supervision of Mr. M. J. Turner. He was a very competent engineer with a background in applied mathematics, and several years of experience with Boeing. The job that Jon Turner had for me was the analysis of the vibration properties of a fairly large model of a 'delta' wing structure that had been fabricated in the Boeing shop. This problem was quite different from the analysis of a typical wing structure which could be done using standard beam theory, and I spent the summer of 1952 trying to formulate a mathematical model of the delta wing representing it as an assemblage of typical 1D beam components. The results I was able to obtain by the end of the summer were very disappointing, and I was quite discouraged when I went to say goodbye to my boss, Jon Turner. But he suggested that I come back in Summer 1953. In this new effort to evaluate the vibration properties of a delta wing model, he suggested I should formulate the mathematical model as an assemblage of 2D plate elements interconnected at their corners. With this suggestion, Jon had essentially defined the concept of the finite element method.*
>
> *"So I began my work in summer 1953 developing in-plane stiffness matrices for 2D plates with corner connections. I derived these both for rectangular and for triangular plates, but the assembly of triangular plates had great advantages in modeling a delta wing. Moreover, the derivation of the in-plane stiffness of a triangular plate was far simpler than that for a rectangular plate, so very soon I shifted the emphasis of my work to the study of assemblages of triangular plate 'elements', as I called them. With an assemblage of such triangular elements, I was able to get rather good agreement between the results of a mathematical model vibration analysis and those*

> *measured with the physical model in the laboratory. Of special interest was the fact that the calculated results converged toward those of the physical model as the mesh of the triangular elements in the mathematical model was refined."*

While Jon Turner's application for DSM was vibration calculations to facilitate flutter and dynamic analysis, Ray Clough realized that DSM could be applied to stress analysis. In 1960, Clough penned the famous paper titled "Finite Elements for Plane Stress Analysis" which both adapted the DSM method for stress analysis and simultaneously coined the phrase "Finite Element." [4].

Besides the work done by those directly affiliated with Boeing, many others contributed to the development and popularization of today's modern finite element method. In particular, J.H. Argyris, O.C. Zienkiewicz, and E.L. Wilson should be credited with their huge contributions in developing and broadening the scope of the finite element method beyond aerospace applications. References1, 5 and 6 provide comprehensive historical background on the development and evolution of the finite element method.

References 2, 4 and 17 can be considered seminal papers that laid out the foundation of the modern finite element method:

- Reference 2: **M. J. Turner, R. W. Clough, H. C. Martin**, and **L. J. Topp**, "Stiffness and Deflection Analysis of Complex Structures," J. Aero. Sci., 23, pp. 805–824, 1956.
- Reference 4: **R. W. Clough**, "The finite element method in plane stress analysis," Proceedings of the Second ASCE Conference on Electronic Computation, Pittsburgh, PA, 1960
- Reference 17: **J. H. Argyris** and **S. Kelsey**, Energy theorems and structural analysis, Aircraft Engrg., Vols. 26 and 27, Oct. 1954 to May 1955

Of significance is that Argyris was a consultant to Boeing [1] in the early 1950's and continued to collaborate with Boeing well into the 1960's [17]. Both Turner and Topp were Boeing engineers, and Clough and Martin were affiliated with Boeing via the summer faculty program. Therefore, it is evident that Boeing, both inspired, and was directly involved in, the research and development that directly led to today's modern finite element method.

Dr. Rodney Dreisbach, (Boeing STF, Retired 2015) summarized Jon Turner's significance in the FEA development and deployment within Boeing's ATLAS program nicely. He wrote about this in the BCA Structures Core "Life@Structures Blog" on November 14, 2013. His closing paragraph reads:

> *"In guiding the not-so-obvious steps leading up to the creation of the FEA Method, Jon Turner has been recognized as a scientist, an engineer, a mathematician, and an innovator. Furthermore, he was a visionary as exemplified by his continued leadership in addressing more advanced flight vehicles such as advanced composite structures for a Mach 2.7 supersonic cruise arrow-wing configuration in 1976, and his continued support and advocacy of Boeing's development of the integrated multidisciplinary structural design and analysis system called ATLAS. The ATLAS System was a large-scale finite-element-based computing system for linear and nonlinear, metallic and composite, structural optimization, including the ply stackup of advanced composite structures. The engineering disciplines represented by the System included statics, weights, dynamics, buckling, vibrations, aeroelasticity, flutter, structural optimization, substructuring, acoustics, nonlinear mechanics, and damage tolerance. Its architecture was comprised of separate modules for the various technical disciplines, all of which shared a common data management system. The System also included several advanced matrix equation solvers and eigensolvers, as well as state-of-the-art substructuring techniques. Substructured interactions could be considered as being static, or as dynamic using either a modal synthesis or branch modes approach."*

Of significance in the above description of ATLAS, is that it closely describes NASTRAN as well. This is not a coincidence. The roots of both NASTRAN and ATLAS date back to the mid–late 1960's. Boeing was the industrial center of finite element analysis and was ahead of the other major aerospace companies in recognizing the superiority of the displacement method and deploying that method within Boeing's precursors to ATLAS.

In 1964, NASA recognized that the future of structural analysis, particularly for complex aerospace structures, was the finite element method. At this time, NASA created a committee composed of representatives from each NASA center and chaired by Tom Butler (considered by Dr. Richard MacNeal to be the Father of NASTRAN). The committee was commissioned to investigate the state of analysis in the aerospace industry and to find an existing finite element program worth recommending to all NASA centers. The first committee action was to visit the aircraft companies that had done prominent work in finite element analysis. In the end, this committee concluded that no single computer program "incorporated enough of the best state of the finite element art to satisfy the committees hopes" and recommended that NASA sponsor development of its own finite element program [18]. This program would be called NASTRAN which is an acronym for NAsa STRuctural ANalysis.

In July, 1965, NASA issued the RFP for NASTRAN. The MacNeal-Schwendler Corporation (MSC) was not recognized as a significant or large enough entity in the finite element world, and so it partnered with Computer Sciences Corporation as the lead in its response to the RFP. Boeing considered the RFP, but in the end did not submit a proposal.

Had Boeing participated, according to Dr. MacNeal (co-founder of the MacNeal-Schwendler corporation), the NASTRAN contract would have certainly gone to Boeing since Boeing was the clear industrial leader in the finite element method.

In the mid-to-late 1990's, as an employee of MSC, the author brought Dr. MacNeal to Boeing's Renton engineering facility where Dr. MacNeal spoke to BCA's team of finite element analysis experts. Dr. MacNeal began his talk by thanking Boeing for not participating in the NASTRAN RFP, and he went on to tell the story of how MSC essentially won the eventual NASTRAN contract due to Boeing's decision to not participate.

Dr. MacNeal writes that Boeing departed NASA's NASTRAN Bidders' Conference after being told that they could not have an exception to NASA's requirement that all work be done on NASA's computers [18]. The NASA purchasing agent, Bill Doles, said that an exception could not be granted because NASA had determined that their computers had a lot of excess capacity and it would be uneconomical to pay the contractors for use of their computers. Boeing responded that they would carry the costs of their own computers as overhead and not charge NASA. Bill Doles responded that this was unacceptable since most of Boeing's work was with the government, and the government would have to pay the overhead anyway. After this exchange, at the next break, the Boeing team abruptly departed the conference.

Nonetheless, Boeing had likely influenced the RFP. The RFP was essentially a collection of what NASA perceived to be the state of the art in FEA that it gathered from its studies of the various aerospace FEA codes. The fact that NASTRAN, (developed according to the requirements of the RFP), both architecturally and capability-wise are closely paralleled by ATLAS may not be due to pure coincidence, but perhaps due to the NASA incorporating Boeing's "state of the finite element art" into the RFP.

III. Craig-Bampton Component Mode Reduction and Synthesis

As mentioned in the last section, ATLAS included several "state-of-the-art substructuring techniques." One of these techniques was Component Mode Reduction. Component Mode Reduction is a technique for reducing a finite element model of a component down to a set of boundary matrices that approximately represent the dynamic characteristics of the component. The accuracy of the approximation is generally improved by increasing the number of component modes retained during the reduction process. The reduced component is generically referred to as a substructure but currently the term *superelement*, coined by commercial FEA software providers, is more prevalent.

There are a litany of component mode reduction and reduced order modeling techniques, but one technique stands out due to widespread usage and deployment in the popular commercial FEA packages (for example, MSC Nastran, NX Nastran, ABAQUS and ANSYS). This technique is the "Craig-Bampton" (C-B) component mode reduction method and this method is applied to a wide variety of dynamic simulations not only in aerospace, where it was conceived, but also in virtually every industry where structural dynamics has a large influence on the product design and performance, especially the automotive industry. [19]

Within Boeing, the C-B technique is central to the Boeing Aeroelastic Process (BAP) that is used for flight loads and flutter analysis. Of significant importance to the flutter community is that the C-B methodology enables rapid frequency variation studies as well as insertion and tailoring of assumed modes. The C-B method is also extensively applied in propulsion dynamics for Windmilling, Fan Blade Out (FBO) loads and Engine Vibration Related Noise (EVRN) analyses.

The EVRN analysis is a coupled vibro-acoustic analysis where C-B reduction is performed on both the airframe and acoustic fluid model and reduced down to the interface with the engine. Of significance, is that this C-B superelement package can be delivered to the engine manufacturers in the form of boundary matrices and output transformation matrices (OTMs), thereby preserving all Boeing IP, while enabling the engine companies to determine how different engine bearing and mount designs effect the interior cabin noise.

C-B reduction with OTMs is also central to Coupled Loads Analysis in Boeing's Space spacecraft industry. Coupled Loads Analysis, in this context, is essentially the dynamic structural analysis of the complete space structure. For example, in the case of a rocket or launch vehicle, you also have the cargo (for example, a satellite). The various components of the launch vehicle and cargo are frequently built by different companies neither company can generally have visibility of the other's finite element models. However, the dynamics of the entire system must be analyzed. This is facilitated by use of superelements typically created using C-B reduction and OTM's similar to what was described with the propulsion EVRN analysis. This process enables all parties to generate the detailed data necessary to analyze and design their structure while preserving any IP, export, and ITAR data requirements.

Outside of Boeing, it was summarized in the Introduction how the automotive industry applies FEA with C-B reduction to their NVH dynamic analyses of their vehicles and sub-systems. Another class of dynamic analysis performed in the automotive industry, and across virtually every other industry (including aerospace) that analyzes dynamic systems is Multi-Body Dynamic (MBD) simulation.

MBD is a numerical simulation method in which systems are composed as assemblies of rigid and/or elastic bodies. Connections between the bodies are modeled with kinematic joints or linear/nonlinear springs/bushings/dampers. If inertia (mass) is eliminated, and all bodies are rigid links with kinematic constraints, then the multibody analysis reduces down to a kinematic mechanism analysis. However, when mass is included, the analysis is inherently dynamic.

For the dynamic case with flexible bodies, the challenge is to bring the flexibility of each body into the system simulation in an accurate and efficient manner. The standard methodology used to create the "flex body" is to perform a C-B reduction where the body is reduced down to the interface DOFs that connect the body to its surrounding joints. Additional transformations may be done to put the interface matrices in a form compatible with the formulation of the MBD software system. However, the first step is typically the C-B reduction. All the popular commercial finite element packages have the ability to generate "flex bodies" of components from finite element models of the component and the C-B method is used to create the reduced mass and stiffness matrices that are processed to generate the flexible body. (There are other techniques beyond C-B that can be used to generate flex bodies, particularly when nonlinearities of the component model are needed. However, for the linear cases most prevalent today, the C-B method is pervasive.)

Therefore, at this point, we have seen that Boeing had a role with the inspiration and development of the finite element method, and we have discussed how the C-B reduction technique is prevalent across industries performing dynamic structural analysis. The C-B reduction technique was also one of the "state-of-the-art substructuring techniques" present in Atlas.

The seminal paper on the C-B method was published as "Coupling of Substructures for Dynamic Analysis" in July 1968 in the AIAA Journal [9] by Roy Craig of the University of Texas and Mervyn Bampton, a Boeing Sr. Structures Engineer. Hence the name of the method "Craig-Bampton."

Figure 2: Roy Craig

Of note is that [9] describes both the C-B reduction technique and synthesis of the multiple C-B reduced parts to generate an accurate system level dynamic model of substantially reduced order enabling both accurate and efficient calculation of dynamic characteristics of highly coupled structures. This AIAA paper has more than 1100 subsequent journal citations since publication demonstrating the impact the C-B methodology on subsequent applications and research. Of course, the motivation for this development within Boeing and Atlas was for application of Flutter and Coupled Dynamic Loads analysis of highly redundant space vehicle and airframe structures.

Also of note is that this very same paper was earlier published within Boeing in 1966 as document D6-15509 [10]. (This document is available electronically from library.web.boeing.com.) This document was prepared by R. R. Craig, supervised by M. C. C. Bampton and approved by L.D. Richmond. This work took place when Craig was employed by Boeing as part of the summer faculty program. [19]

Therefore, just as we saw substantial collaboration between Boeing and leading researchers in the development of the finite element method, we see a similar collaboration with Roy Craig in inspiration, development, and deployment of the Craig-Bampton method for Boeing's dynamic analysis needs. The methodology is most credited to Roy Craig who spent his 40 years at the University of Texas specializing in development of computational and experimental methods of flexible substructures. However, the need by Boeing for efficient and accurate coupled dynamic analysis methods inspired and accelerated the development that became the Craig-Bampton technique and 50 years later, this Craig-Bampton method is omnipresent! [19]

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IV. THE LANCZOS METHOD OF EIGENVALUE EXTRACTION

The natural frequencies of a structure may be the most fundamental dynamic characteristic of a structure. Dynamicists use the natural frequencies and associated mode shapes to understand dynamic behavior and interplay of components in a dynamic system. The computation of a structure's or substructure's natural frequencies and mode shapes is of fundamental importance to the dynamicist.

From a mathematical perspective, the calculation of natural frequencies and mode shapes is an eigenvalue extraction problem in which the roots (eigenvalues) and associated mode shapes (eigenvectors) are computed from the dynamic equation of motion with the assumption of harmonic motion while neglecting damping and applying no loading.

Eigenvalue/Eigenvector calculation is also a requirement of the C-B reduction method. The C-B method uses the natural frequencies and mode shapes of a component constrained at its interface to generate the dynamic portion of the reduced stiffness, mass and loads matrices. Therefore, a robust and efficient C-B reduction requires a robust and efficient eigenvalue/eigenvector calculation.

The Lanczos eigenvalue extraction method is by far the most prevalent eigenvalue extraction method used today in the popular finite element programs for vibration and buckling modes. Today, the AMLS and ACMS methods are promoted as the state-of-the-art eigenvalue/extraction methods for the largest models commonplace in the automotive industry.

While AMLS and ACMS can easily outperform the Lanczos method on large models, they are essentially automated methods of substructuring the mathematical finite element model utilizing enhanced C-B reduction for an accurate approximation of each substructure. When these C-B reduced substructures are assembled, a final system level eigenvalue extraction is performed to compute approximate system level modes.

This complete substructuring, assembly, and solution process is captured in the AMLS and ACMS methods. However, it is typically the Lanczos method with C-B reduction that is utilized to form the reduced approximate system that was solved to obtain the approximate system frequencies and mode shapes.

The Lanczos method is the bread and butter of dynamicists, whether used directly for computation of natural frequencies and mode shapes, or used indirectly with the AMLS/ACMS and similar methods that are based upon automated component modal synthesis of very large systems.

Prior to the commercial availability of the Lanczos method in the mid 1980's, dynamicists spent a large amount of thought and time in determining how to reduce a model down to a size that could be efficiently solved with their finite element program and yield an accurate, albeit approximate solution. This is precisely why the C-B and other dynamic reduction techniques were created. However, the underlying weakness of all these methods was an accurate, efficient, and robust eigenvalue extraction method for the reduction process.

From a high level, there were essentially two families of eigenvalue extraction methods from which a dynamicist could choose: 1) Iterative based methods such as Inverse Power and Subspace Iteration, and 2) Tridiagonal methods such as the Householder-QR method. The iterative methods were relatively fast and efficient, but suffered from accuracy issues and struggled with closely spaced and large numbers of roots. The Tridiagonal methods were relatively robust and could accurately solve for all the roots of a system. Unfortunately, they also required enormous amounts of memory and were very inefficient making them impractical for all but the smallest models. The Lanczos method gained instant popularity because it could solve large models both accurately and efficiently, eliminating the tedious reduction process for a large variety of dynamic analyses.

In the 1960's-1980's substructuring and component mode reduction were primarily performed to enable computation of a system's modes when the system could not be solved without reduction on the computers of the time due to memory, disk, and time constraints. After the commercial availability of the Lanczos method, substructuring and component mode reduction were primarily performed for other reasons, such as to enable efficient frequency variation studies (as is the case with BCA's standard flutter analysis process), or to generate reduced matrix level models of components that can be shared with a third party to assemble into their system.

Only in the last 15 years with the advent of High Performance Computing (HPC) systems, have the AMLS/ACMS methods brought us back to substructuring as the norm for solving the largest eigenvalue problems because parallelization and improved performance is more easily enabled using a substructured solution process.

So what does Boeing have to do with the Lanczos method? It is twofold. First, the method was invented by Cornelius Lanczos. He published the method in 1950 while working at the National Bureau of Standards [11, 14]. However, prior to joining the National Bureau of Standards, Lanczos was employed with the Boeing Aircraft Company in Seattle from 1946-49 where he was inspired to study and improve matrix methods and numerical eigenvalue extraction of linear systems. Shortly after leaving Boeing, he completed the formulation of what we now call the Lanczos eigenvalue extraction method [12, 13].

Cornelius Lanczos was a colleague of Albert Einstein, and on December 22, 1945, he penned this passage in a letter to Einstein:

> *"In the meantime, my unfavorable situation here at the University has changed for the better. I have been in cooperation with Boeing Aircraft in Seattle, Washington for almost two years. Our relationship has developed in such a way that the company offered me a permanent position. It is somewhat paradoxical that I with my scientific interest can always get on as an applied mathematician." [13]*

Figure 3: Cornelius Lanczos at his desk at Boeing Plant 1, Seattle, WA

More insight into Lanczos' inspiration from his tenure at Boeing is obtained in his recorded interview by the University of Manchester in 1972 [12]. There are several references to his time at Boeing where among other things, he mentions:

> *"of course this eigenvalue problem interested me a great deal because in Boeing one encountered this eigenvalue problem all the time and the traditional methods, they give you – it was easy enough to get asymptotically the highest eigenvalue, but the question is how do you get all the eigenvalues and eigenvectors of a matrix in such a way that you shouldn't lose accuracy as you go to the lower eigenvalues… I knew of course from theoretical physics that eigenvalues and eigenvectors, I mean wave mechanics, everything, is eigenvalues and eigenvectors. Only in this case it was numerical, and in Boeing when I was frequently asked to give lectures, one of the lecture topics was matrices and eigenvalues and linear systems so that I was familiar in a theoretical way of the behavior of linear systems, particularly large linear systems."*

After joining the National Bureau of Standards, Lanczos had the opportunity to complete the formulation of his method based upon his experience at Boeing. He applied it on an analog computer available to him, but in the end, he doubted the practicality of his method. In reference 12, he tells this story:

> *"And I will never forget when I think it was an 8x8 matrix and the eigenvalues varied in something like 10^6. I mean the highest to the lowest, and I expected that the highest eigenvalues would come out to 10 decimal places and then*

> *we gradually lose accuracy but actually all the eigenvalues came out to 10 decimal places. I mean this was a tremendous thrill to see that, that we didn't lose anything, but of course it had to require the careful reorthogonalization process which makes my method practically, let's say, of less value or perhaps even of no value."*

It is somewhat entertaining that the roots of the de facto standard eigenvalue extraction method for nearly 30 years were thought by its inventor to be "of less value, or perhaps even no value." Of course, by Lanczos' own admission, the method was difficult to apply in practice. However, the significance of the Lanczos method in maintaining accuracy was not lost on the mathematical community and over the years, many mathematicians studied the method and searched for numerical methodologies that would make the method practical and of high value. An in-depth historical development of the Lanczos method is beyond the scope of this writing. However, this leads us to Boeing's second point of influence on the Lanczos method: The development and deployment of the first robust commercially viable implementation of the Lanczos method.

V. BOEING COMPUTER SERVICES AND BCSLIB

The late 1960's is a significant period for Boeing as well as for finite element analysis, numerical computing, and mainframe computing data centers. At this time, Boeing had just launched the 747 in 1969 and was about to enter the big "Boeing Bust" which saw its employment drop from >100,000 down to under 40,000 by the end of 1971. At the same time, within Boeing, this bust is perhaps responsible for the consolidation of two largely disconnected Boeing math groups: one on the military side and one on the commercial side. In 1970, Boeing Computer Services (BCS) was formed and these two math groups were brought together under the BCS organization [15].

By the 1980's, BCS had both a mature data center where time was leased on Boeing computers to run commercial applications like NASTRAN and ANSYS. The expertise of the math group resulted in the establishment of a software group that built and licensed the math library BCSLIB-ext as well as developed the systems and controls software Easy5 (the "-ext" version of BCSLLIB was licensed externally. BCSLIB was used internally).

During the 1980's and early 1990's the BCS math/software team had a major impact on solutions of large linear static and dynamic systems. Notably, they were directly responsible for the first significant robust and efficient Lanczos method deployed in a commercial FEA package. In 1985, The MacNeal-Schwendler Corporation (MSC) released Nastran V65 with Boeing's Lanczos eigensolver [22] and in the decade following, similar implementations were deployed in most of the other popular finite element packages.

The major players on the Boeing side were John Lewis, Horst Simon, Roger Grimes, and their manager Al Erisman. Louis Komzsik, from MSC also played a major role. Louis recognized the impact the Lanczos method would have if implemented robustly. He convinced MSC to fund Boeing to bring the Lanczos method to fruition in MSC Nastran. Louis was a perfectionist and drove the Boeing team to handle everything that could break so as to make it as bomb-proof as possible.


Figure 4: John Lewis, Horst Simon and Roger Grimes


Figure 5: Louis Komzsik

Taking the Lanczos method from an unstable, impractical methodology to a highly practical, robust and efficient methodology was the result a many researchers and the coalescence of several key break-throughs. The summary, as provided to the author during an interview with John Lewis in May 2016 is as follows: The Lanczos algorithm in Boeing's BCSLIB code combined work from five PhD theses with critical industrial support. The key contributions to efficiency are:

1) Block algorithms (2 Stanford PhD theses – Richard Underwood, John Lewis))
2) Stability correction only as needed (2 Berkeley PhD theses – David Scott, Horst Simon, part of one of the Stanford theses -- Lewis)
3) Shifting (Swedish PhD thesis – Thomas Ericsson)
4) Integrating all of these with a smart algorithm for choosing shifts (BCS – Grimes, Lewis & Simon)

The "creative break through," according to John Lewis, emerged over a couple of pints with David Scott while in a pub in Reading, England in 1980 where they discussed Ericsson's work on shifting and came up with a plan to improve upon earlier Lanczos method implementations. However, they could not get funding to implement the plan, so it sat for several years. In John Lewis's words, Louis Komzsik emerged as the "Guardian Angel" when he brought forward the funding from MSC to implement the plan in MSC Nastran. Louis was Hungarian as was Lanczos, so he had great faith in his countryman's idea!

Besides the Lanczos component, the other major thrust of BCSLIB was the sparse direct linear equation solver. This solver provided a substantial performance boost in solution of large linear systems and played a significant role in Lanczos performance. Within the Lanczos implementation, a series of linear solutions (matrix decompositions) takes place as the algorithm searches for the eigenvalues. Maximum performance is dependent on minimizing the number of decompositions. This requires algorithms for selection of good trial eigenvalues along with transformations to find both closely space roots and widely separated roots efficiently. (What is described here is the "shifting" and "smart algorithm for choosing shifts" mentioned above.)

The work of the BCS math team cannot be overstated when it is recognized that 30–35 years after their heroic efforts, the Lanczos method is still prominent in the popular commercial FEA packages. We attribute much of the performance improvement in finite element solutions to computing hardware improvements. However, in the late 1980's, between the Lanczos and Sparse Solver methods, engineers realized order of magnitude gains in solution performance independent of any hardware improvements. These two performance improvements meant that many models that had previously required substantial substructuring and complex dynamic reduction could now be solved directly with the Lanczos method.

Also of significance is that this Boeing team, along with Cray went on to win the 1989 Society of Industrial and Applied Mathematics (SIAM) Gordon Bell Award. They received their award specifically for achieving record performance with their implementation of a general sparse matrix factorization on an 8-processor Cray Y-MP computer. This Sparse Matrix Solver development was another great effort that found its way into the commercial FEA codes that contributed to both Lanczos efficiency and solution efficiency of linear systems of equations.

In closing this section, the overall contribution Al Erisman made, should be noted. Erisman managed and directed the math group from 1975 until his retirement in 2001. According to John Lewis, "Al Erisman created the ethos of the Boeing Math Group, which strongly valued academic-industrial collaboration." Were it not for Erisman, the industrial collaboration between MSC and Boeing may never have taken place.

VI. CONCLUSION

The finite element method was invented roughly 60 years ago. Craig-Bampton reduction was invented roughly 50 years ago and the modern Lanczos and Sparse solver methods were deployed into commercial FEA packages roughly 30 years ago. Virtually every Boeing product created since the 1950's relied significantly in whole or in part on these technologies. The same can be said outside of Boeing where multitudes of consumer products ranging from toys to automobiles are engineered with significant application of these technologies. In many cases, engineers are utilizing these technologies today within modern GUI's with no idea of the underlying solution methods and algorithms at play. The fact that after multiple decades, these technologies persist, albeit in often simpler and automated implementations, is a testament to the significance of these methods. Moreover, while Boeing did not solely invent any of these technologies, Boeing's need to engineer some of the most complex and high performance structures, had a tremendous influence on the development and eventual deployment of these methods. We feel and see the effects of these technologies in the products all around us today. As we celebrate Boeing's Centennial, it is appropriate to not only applaud our predecessors for the impact the products they engineered had on our society, but also applaud the engineers and mathematicians at Boeing who contributed to solution methods and algorithms that are routinely applied outside of Boeing to the development of the superb products that grace our society today.

It is also fitting to mention that on the same day this conclusion was penned, the author received an assignment to generate reduced dynamic models for the 777-9X folding wing tip. The author will utilize the aeroelastic finite element model along with C-B reduction and Lanczos eigenvalue extraction to form the flexible body representation of the airframe and folding wing tips. These reduced dynamic models will be integrated into the external controls multi-body dynamic system model. Therefore, the work of Boeing engineers/mathematicians Turner, Bampton, Lanczos, Lewis, Simon, and Grimes will be applied to engineer perhaps the most iconic feature of Boeing's next great commercial airplane. However, this is not unusual since as previously mentioned, superb products are being engineered all over the world with exactly these same methods every day!

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